Bio-Inspired SAT

Jan Klátil, Milan Rybář

Outline

- EA and SAT
- General ideas for EA
- Belief propagation
- GPU

Evolutionary algorithm (EA)

- Pseudo-code for a genetic algorithm:

```
Initialize the population
Evaluate initial population
Repeat
    Perform competitive selection
    Apply genetic operators to generate new solutions
    Evaluate solutions in the population
Until some convergence criteria is satisfied
```

EA and SAT - Representation

- Bit String Representation
- Floating Point Representation
  - transforming SAT into a continuous optimization problem
    - replace literals x_j and $\neg x_j$ by $(x_j-1)^2$ and $(x_j+1)^2$
    - substitute $\wedge$ and $\vee$ by * and +
    - minimization
- Path Representation
- Graphs:
  - Node: variables, edge: appear at the same clause
  - Node: clause, edge: share the same variable

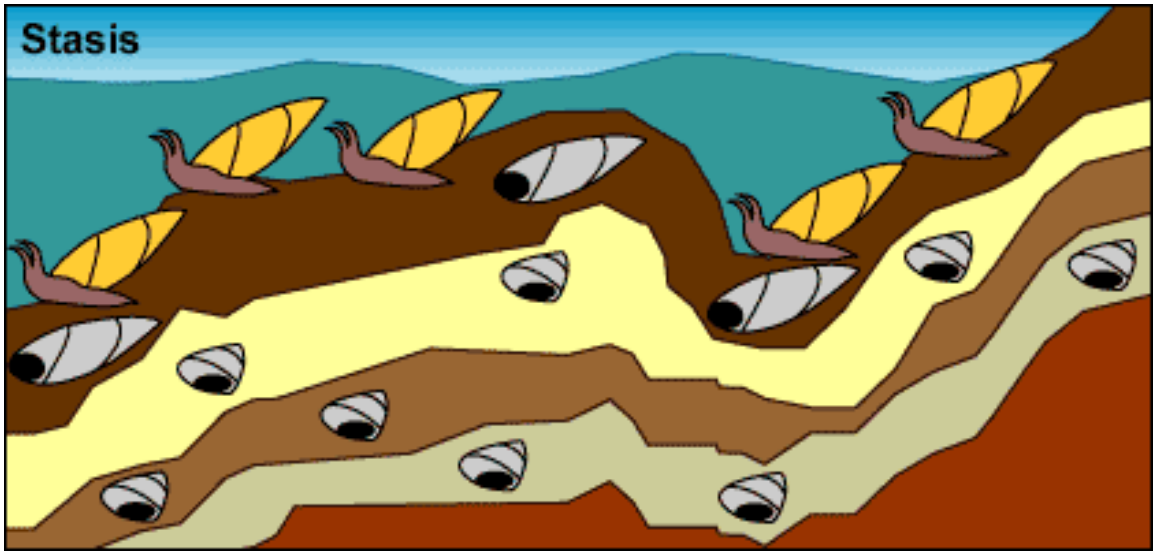
EA and SAT - Operators and Fitness

- Operators
  - Crossover
  - Mutation
- Local search => Memetic algorithms (MA)
- Fitness
  - $f_{MAX} = c_1(x) + \ldots + c_m(x)$
  - $f_{SAW}(x) = w_1 * c_1(x) + \ldots + w_m * c_m(x)$
    - stepwise adaptation of weights (SAW) principle
    - $w_i \leftarrow w_i + 1 - c_i(x^*)$, x^* is the current fittest individual
  - $f_{REF}(x) = c_1(x) + \ldots + c_m(x) + \alpha * r(x)$, $\alpha \in [0, 1)$
    - refining function
  - Multi-objective fitness

General ideas for EA

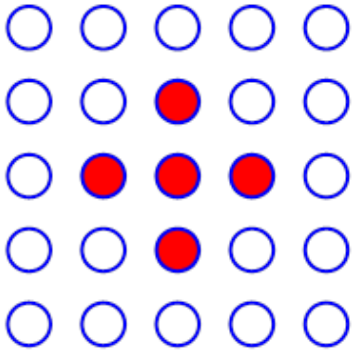
- Structured-population EA
  - Standard GA = Panmictic model of GA
- Multi-agent EA
- PSO and its variants

Island GA



- Populate islands
- While (not :-))
  - Separate island evolution
  - Replacement of individuals

Cellular EA



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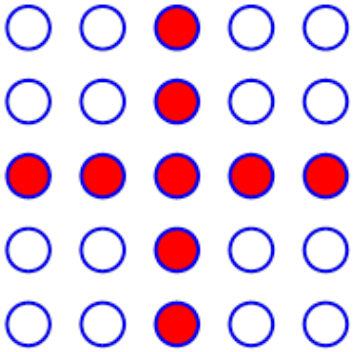

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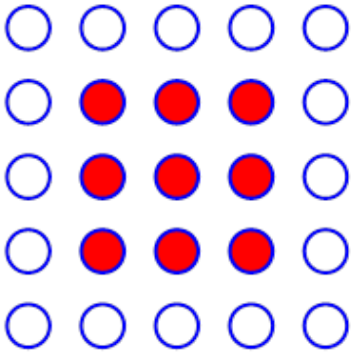

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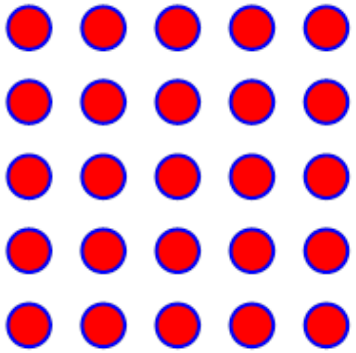

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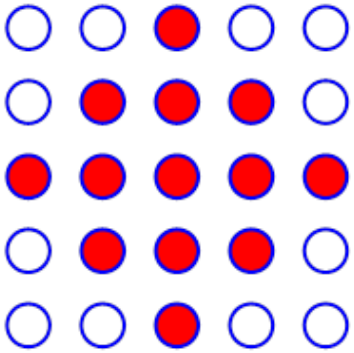

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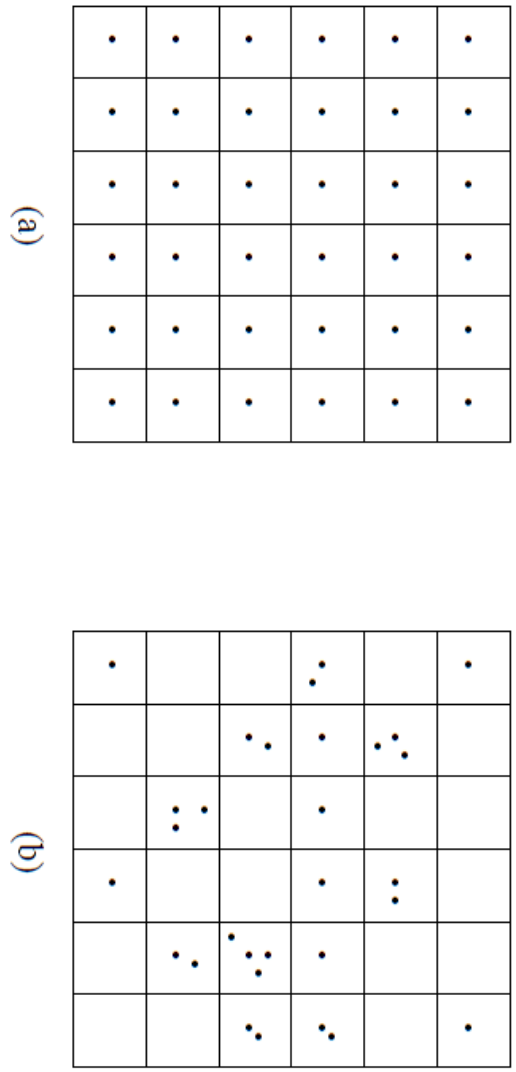
Patchwork GA



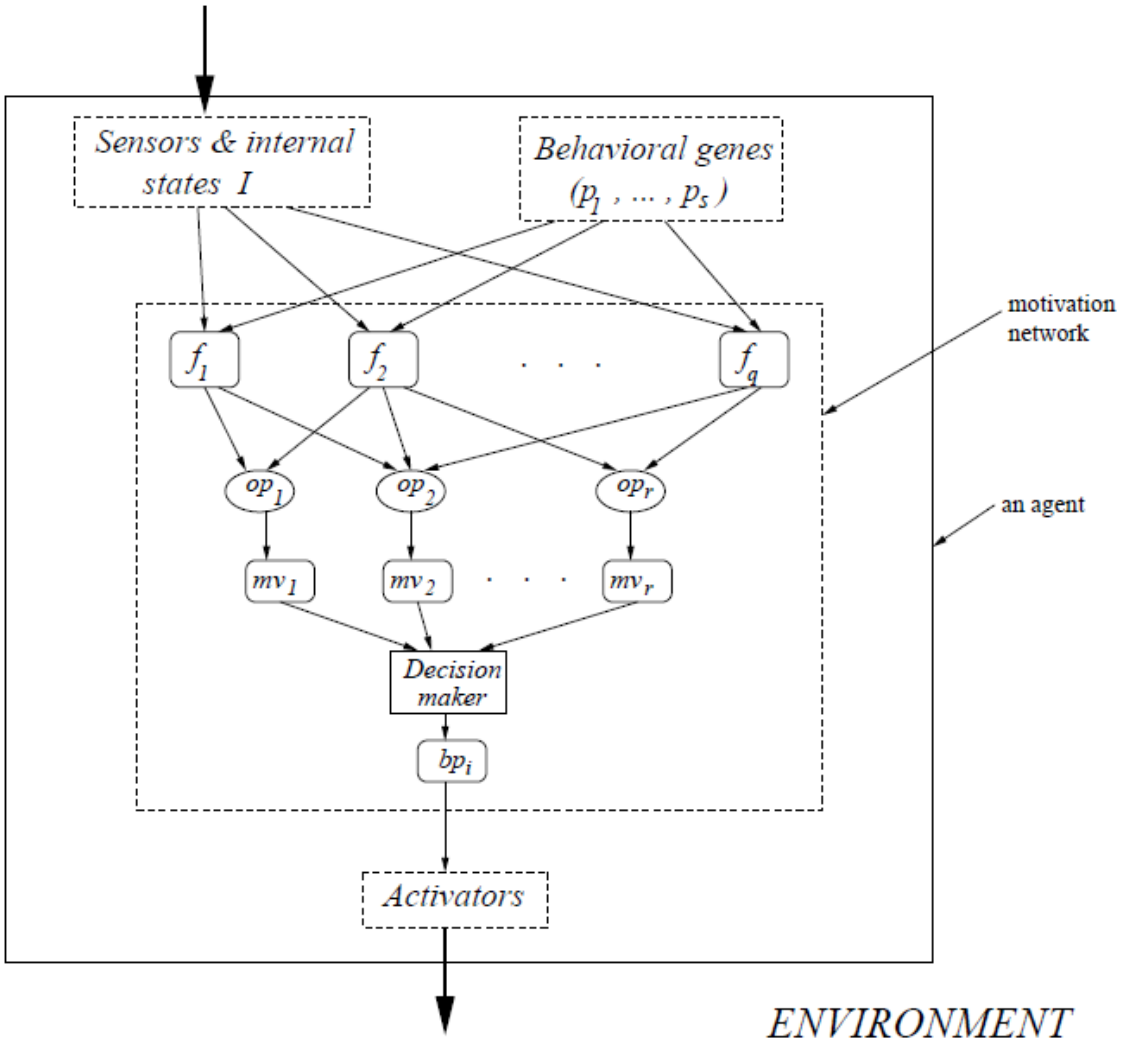


Figure 1: A grid interconnection topology in classical diffusion models (a) and in the PATCHWORK model (b)

Terrain-Based GA

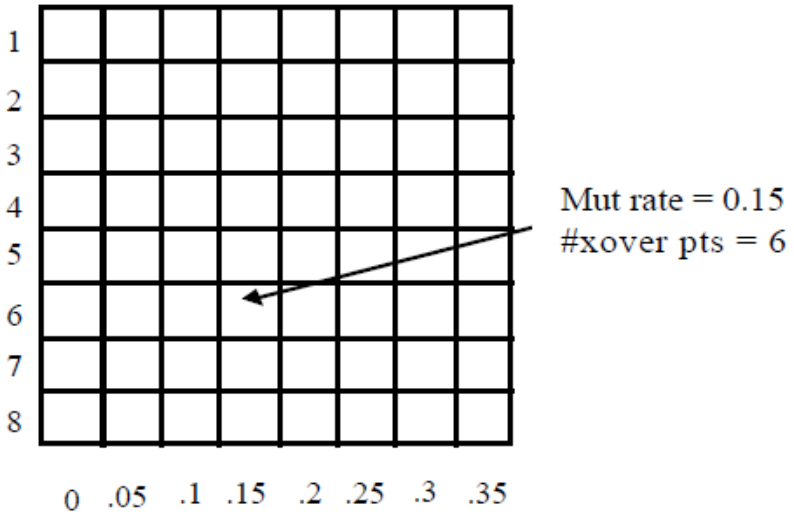


Figure 2: *Mutation rate* Spread Along the X-axis of a CGA, *Number of Crossover pts* Spread Along the Y-axis.

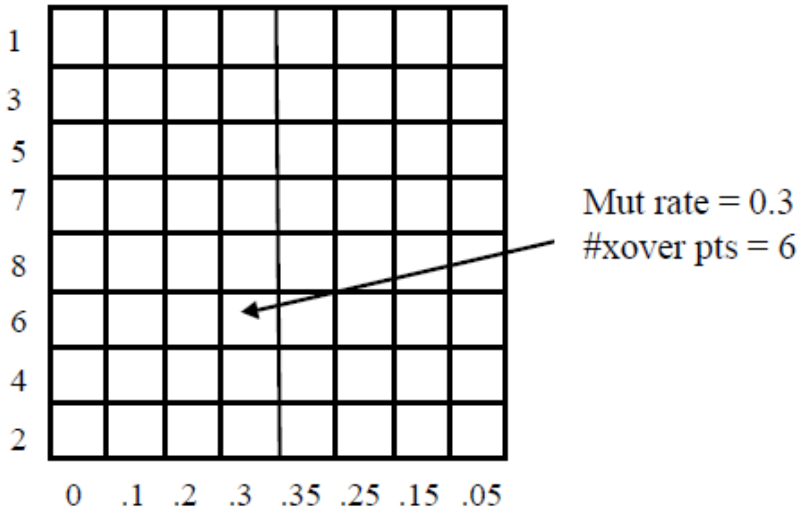
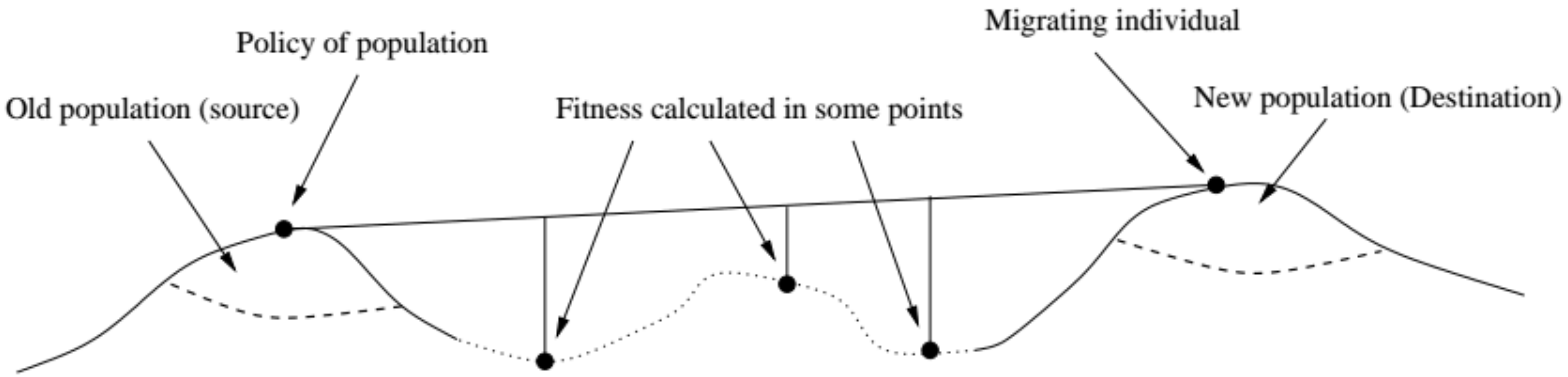


Figure 3: *Mutation Rate* (along the X-axis) and *Number of Crossover Points* (along the Y-axis), after Sifting

Religion-based GA

- Initialization
- While(not :-))
  - Random Walk
  - Conversion
  - Mating

Multi-national GA



Multi-agent EA

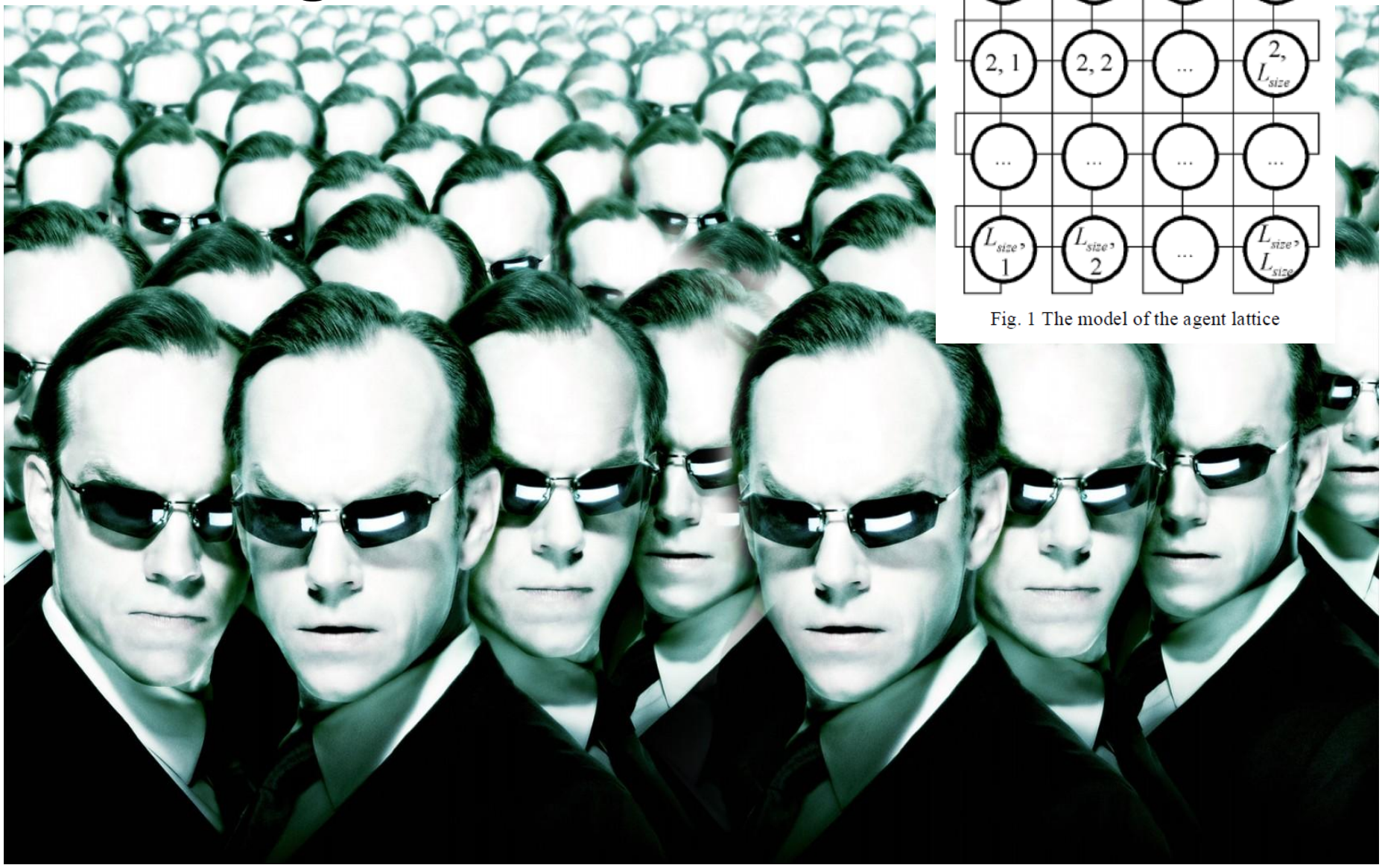
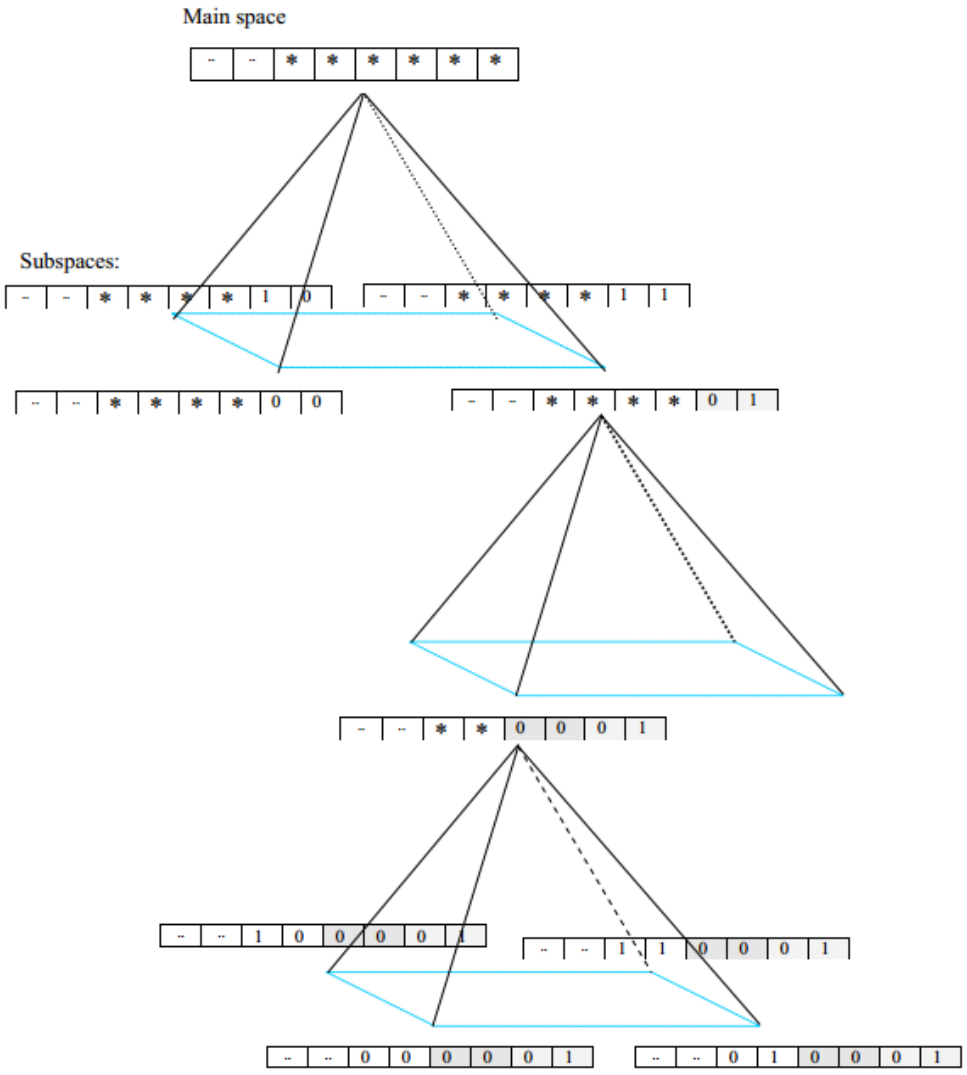


Fig. 1 The model of the agent lattice

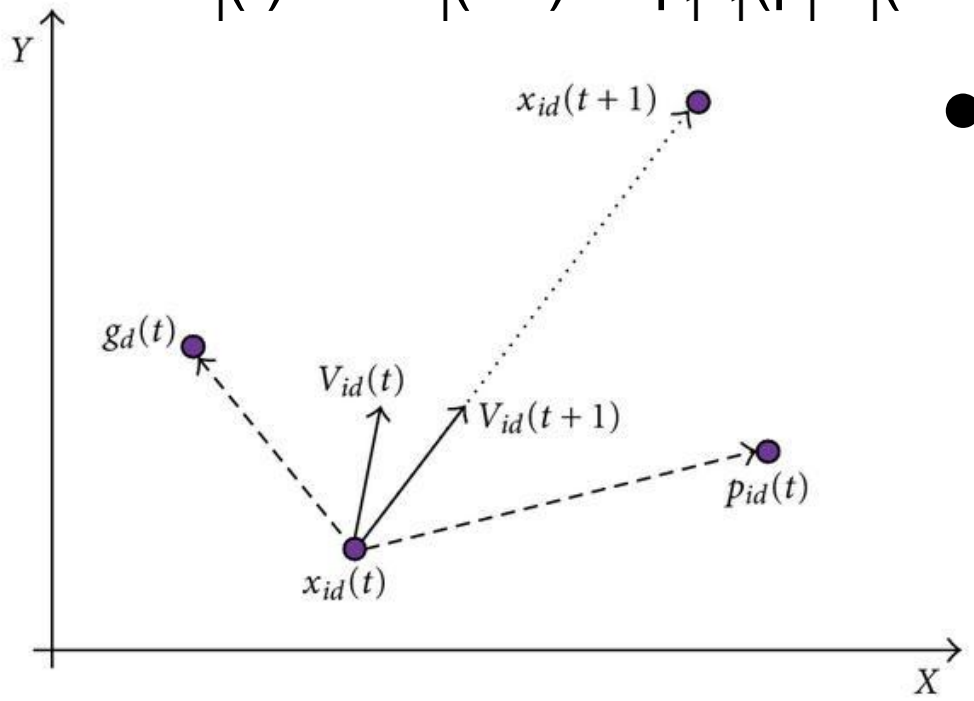
Hypercube Tree

- Initialization
- While(not :-))
  - Selection and mating in leaf and with neighbours
  - Split leaf in hypercubes?
  - Join leaves?



Particle Swarm Optimization (PSO)

- Inspired by the social behavior of bird flocking
- $x_i(t) = x_i(t-1) + v_i(t)$
- $v_i(t) = \omega v_i(t-1) + \varphi_1 r_1(p_i - x_i(t-1)) + \varphi_2 r_2(p_g - x_i(t-1))$



- Parameter selection:
  - maximum velocity
  - acceleration constants
  - inertia constant

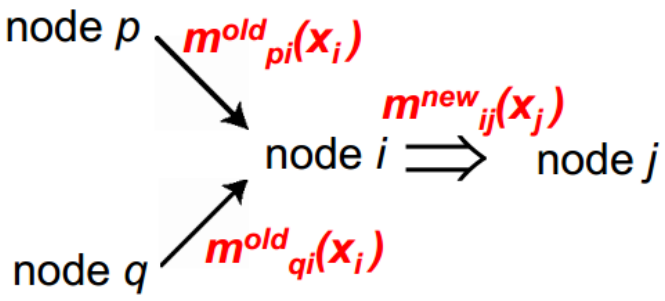
PSO - Variants

- Topology:
  - global best
  - local best
- Binary PSO
  - $s(v_i(t)) = 1 / (1 + \exp(-v_i(t))$ *(sigmoid function)*
  - $x_i(t) = 1$, if rand < $s(v_i(t))$
  - $x_i(t) = 0$, otherwise
- Memetic approach (local search)
- Hybrid PSO
  - Genetic Algorithm and PSO
  - Evolutionary Programming and PSO
  - Differential Evolution and PSO
- PSO in Complex Environment
  - Multi-objective Particle Swarm Optimization
  - Constraint Handling in PSO
- Other Variants of PSO
  - Gaussian PSO
  - Dissipative PSO
  - Cooperative PSO
- ...

Belief propagation

“I (variable x) think that you (variable y) belong in these states with various likelihoods…”

- Converges for trees
- Empiric proof for general graph: O(#vars * exp(k))
  - where k=3, 4, 5, .. ?

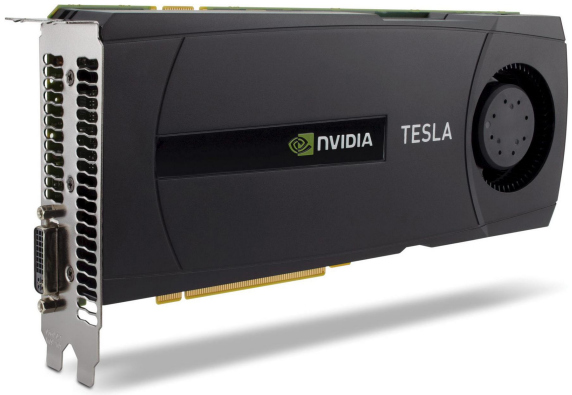


$$m_{ij}^{new}(x_j) = \sum_{x_i} f_{ij}(x_i, x_j) \underbrace{g_i(x_i) \prod_{k \in Nbd(i) \setminus j} m_{ki}^{old}(x_i)}_{h(x_i)}$$

$$m_{ij}^{new}(x_j) = \sum_{x_i} f_{ij}(x_i, x_j) h(x_i)$$

SAT & GPU

- Poor results with WalkSat
- Survey SAT: 20x faster
- MiniSat: 2x faster
- DPLL: 10x faster



Q & A

References 1/2

- *Jens Gottlieb, Elena Marchiori, Claudio Rossi,* **Evolutionary Algorithms for the Satisfiability Problem**, Evolutionary Computation, Spring 2002, Vol. 10, No. 1, Pages 35-50
- *Stefan Harmeling,* **Solving Satisfiability Problems with Genetic Algorithms**, March 9, 2000
- *Lotfi, Nasser, Tamouk Jamshid, Farmanbar Mina*, **3-SAT Problem: A New Memetic-PSO Algorithm**, 06/2013
- *Del Valle, Y.; Venayagamoorthy, G.K.; Mohagheghi, S.; Hernandez, J.-C.; Harley, R.G.,* **Particle Swarm Optimization: Basic Concepts, Variants and Applications in Power Systems**, Evolutionary Computation, IEEE Transactions on , vol.12, no.2, pp.171,195, April 2008
- *Mohamed A. Belal, Mohamed H. Haggag*, **A Structured-Population Genetic-Algorithm based on Hierarchical Hypercube of Genes Expressions**, International Journal of Computer Applications 64 (22): 5-18, February 2013
- *Enrique Alba, José M. Troya*, **A Survey of Parallel Distributed Genetic Algorithms**, Complex. 4, 4 (March 1999), 31-52
- *Enrique Alba, Bernabé Dorronsoro, and Hugo Alfonso*, **Cellular Memetic Algorithms,** JCS&T Vol. 5 No. 4, December 2005
- *V. Scott Gordon, Rebecca Pirie, Adam Wachter, Scottie Sharp*, **Terrain-Based Genetic Algorithm (TBGA): Modeling Parameter Space as Terrain,** Proceedings of the Genetic and Evolutionary Computation Conference, 1, page 229--235
- *Thiemo Krink, Brian H. Mayoh, Zbigniew Michalewicz*, **A PATCHWORK Model for Evolutionary Algorithms with Structured and Variable Size Populations,** 10/1999
- *Jinshu Li, Heyong Wang, Jing Liu, Licheng Jiao*, **Solving SAT Problem with Multiagent Evolutionary Algorithm**, Evolutionary Computation, 2007. CEC 2007. IEEE Congress on , vol., no., pp.1416,1422, 25-28 Sept. 2007

References 2/2

- *Tapabrata Ray and K. M. Liew:* **Society and Civilization: An Optimization Algorithm Based on the Simulation of Social Behavior**
- *Rasmus K. Ursem:* **Multinational evolutionary algorithms**
- *W N Martin, Jens Lienig and James P Cohoon*: **Island (migration) models: evolutionary algorithms based on punctuated equilibria**
- *Rene Thomsen, Peter Rickers and Thiemo Krink*: **A Religion-Based Spatial Model For Evolutionary Algorithms**
- *Gianluigi Folino, Clara Pizzuti, and Giandomenico Spezzano*: **Parallel Hybrid Method for SAT That Couples Genetic Algorithms and Local Search**
- *L. Kroc, A. Sabharwal, B. Selman*: **Satisfied by Message Passing**
- *J. Hao, R. Dorne:* **A new population based method for Satisfiability problem**
- *Alessandro Dal Palu, Agostino Dovier, Andrea Formisano and Enrico Pontelli*: **Exploiting Unexploited Computing Resources for Computational Logics**