Qualitative Reasoning with Time and Space

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What is **qualitative reasoning**?

- Qualitative × quantitative reasoning
- Reasoning about knowledge represented using a limited vocabulary instead of numbers

A means to express conceptual knowledge such as the physical system structure, causality, start and end of processes, assumptions and conditions under which facts are true, qualitatively distinct behavior, etc.

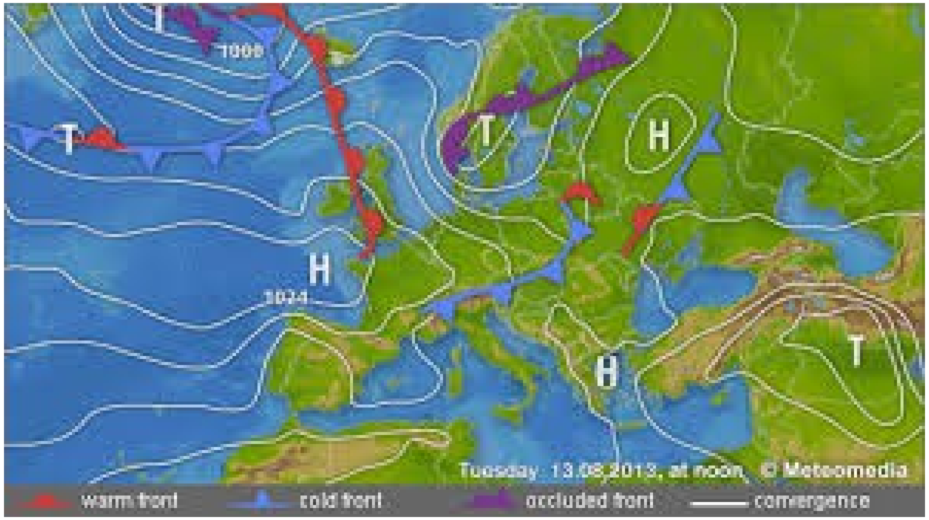
(Bredeweg et. al. 2009)

Principles of QR

- Discrete values
  - Zero/low/high – instead of - numerical value
- Relevant values – use only as many values as needed in the task
  - Outside/inside - instead of - in front of/behind/beside/inside
- Ambiguous values or results - provide a range of answers instead of one
  - Instead of computing exact measure of temperature change
    answer that the temperature is constant or increasing
- Modeling a process - represent the states and transitions between them
  - If the temperature of water is below the boiling point,
    then the water level is constant or slowly decreasing
  - If the temperature of water is above the boiling point,
    then the water level is rapidly decreasing

Examples of qualitative values

- Qualitative categories of numerical values
  - Representation of real values as {+,–, 0}
- Qualitative relationships between entities
  - Higher than, lower than, before, inside of
- Example - expressions about weather:
  - Low/high pressure systems
  - Warm/occluded/cold front

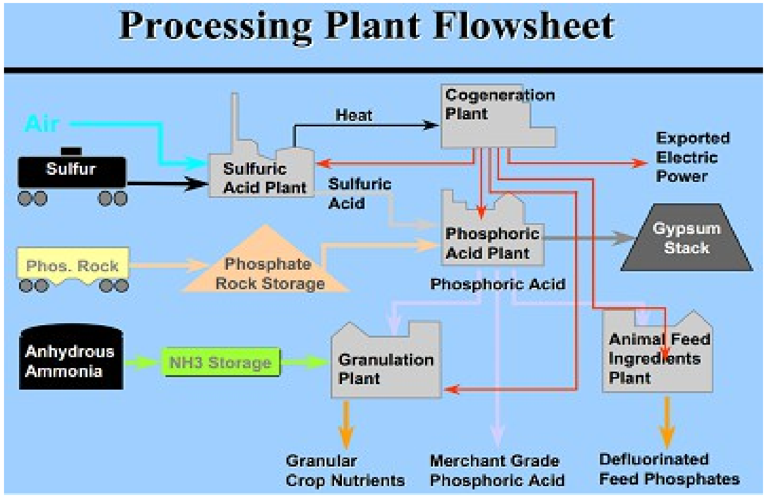


Benefits of QR

- It is considered closer to how humans reason about commonsense knowledge
- Allows inexpensive reasoning about basic properties of entities in the physical world
- It can permit the simulation of complex systems
- It is possible to deal with incomplete knowledge
- Imprecise but correct predictions
- Compact representation of knowledge
  - Qualitative categories make only as many distinctions as necessary

Real-world applications of QR

- Geographic information systems (GIS)
- Robotic navigation
- Natural language understanding
- Monitoring, diagnosis, effect analysis
- System verification, explanation generation
- Simulating qualitative models of physical systems
  - Automobiles
  - Physiology of the body
  - Chemical processing plants
- Computer-aided design



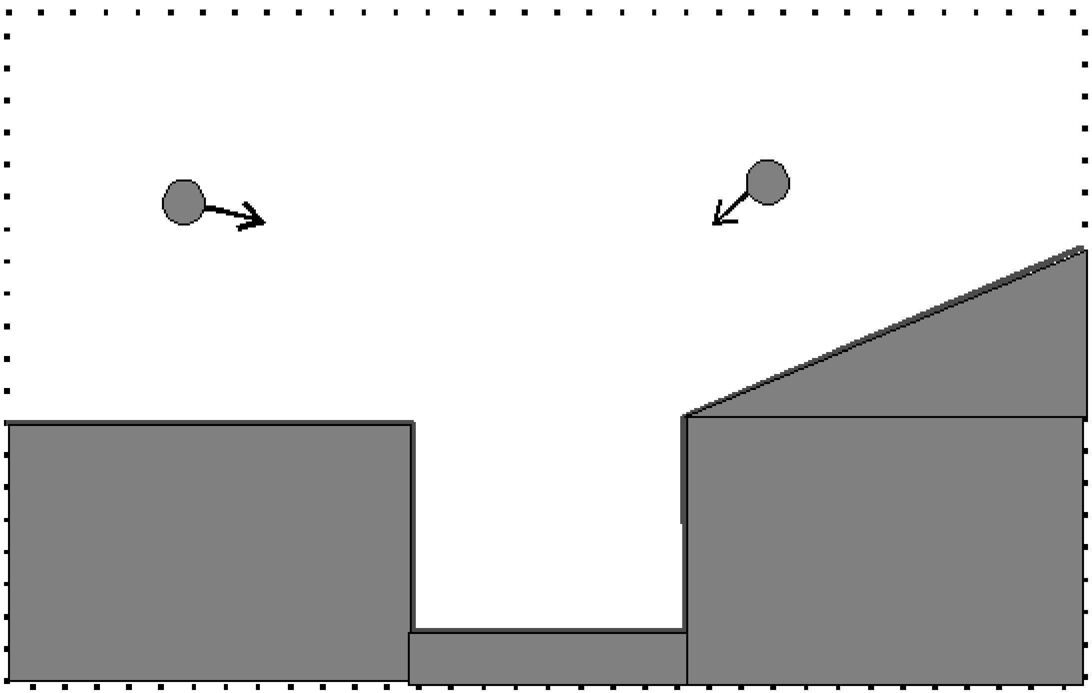
The MD/PV model

- Metric diagram - quantitative information
  - Can support perceptual-like processing
- Place vocabulary - qualitative information
  - Distinctions in shape and space relevant to the current task

Poverty Conjecture: There is no problem-independent, purely qualitative representation of space or shape. (Forbus et al., 1987)

The FROB system

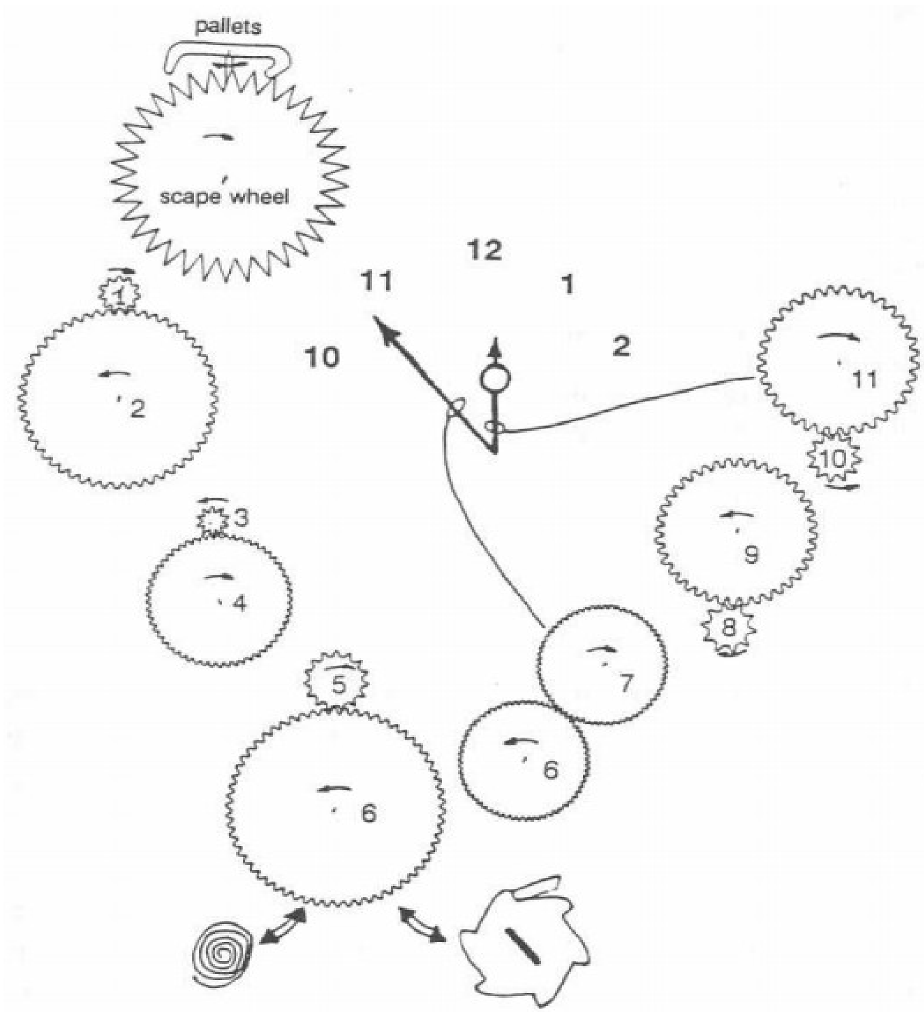
- Reasoning about motion of balls in a 2D diagram
- Estimates about whether two balls might collide
- Two balls thrown into the same well might collide, while the balls thrown into different wells cannot collide

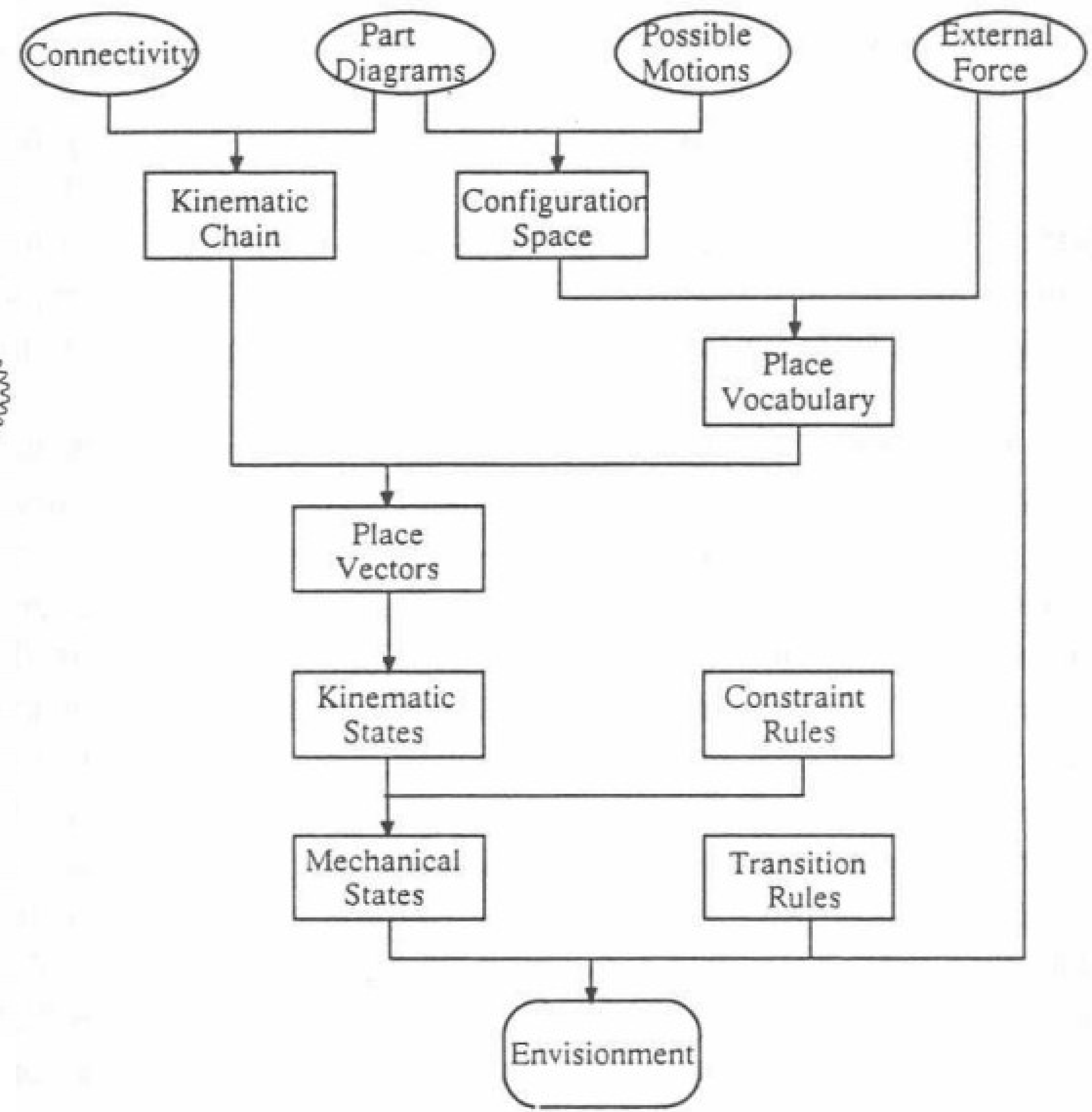


The CLOCK system

- Qualitative simulation of a mechanical clock
- Fixed-axis mechanisms, 2D interactions
- CAD-like description of the parts
- Computes a place vocabulary based on configuration space
- Defines kinematic states for the whole mechanism using elements from these vocabularies
- Qualitative reasoning about the dynamics of motion
  - Uses qualitative vector algebra
- Diagrams usually have noise
  - Has algorithms for removing "small" places and merging places that were "very close"
  - Removing noise can significantly reduce the size of computed place vocabulary

The CLOCK system





Qualitative reasoning

- Most of used calculi can be formalized as abstract relation algebras
- Models are using qualitative values and relations between them
- The set of qualitative binary simple relations is jointly exhaustive and pairwise disjoint set of relations (JEPD)
- The set of all possible relations is the set of all possible unions of the simple relations.
- Reasoning can be done on symbolic level by exploiting composition of relations
- As reasoning methods can be used constraint based techniques
- Important tool is path-consistency algorithm
- Descriptions of compositions of relations are usually stored in a composition table
- Almost all qualitative temporal/spatial calculi are computationally intractable. However, it is possible to identify tractable subsets

Qualitative temporal reasoning

- One-dimensional space
- Temporal expressions mainly describe order and duration
- Common expressions in natural language:
  - before, during, long, a while
- Time can be qualitatively represented by:
  - Time points
  - Intervals

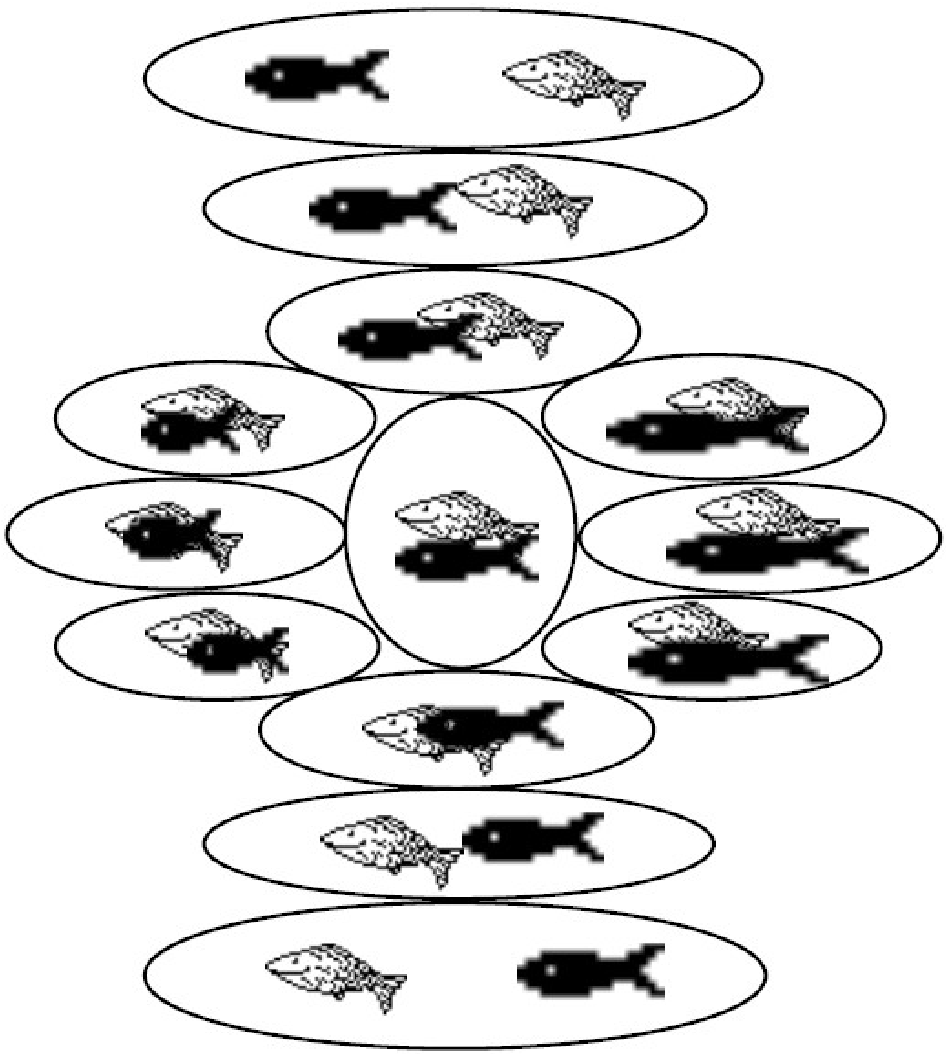
Point algebra

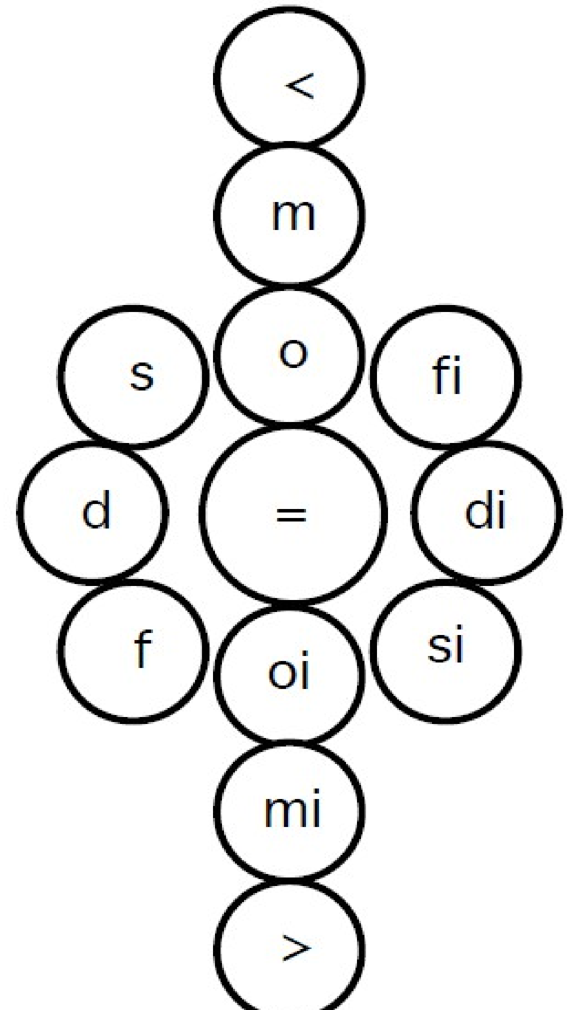
- Represents time as time points and relations between them
- Relations can be decomposed into disjunctions of three simple relations S = {<,=,>}
- All relations: R = { {}, {<}, {=}, {>}, {<,=}, {>,=}, {<,>}, {<,=,>} }
- Operations with relations:
  - Union ∪ - represents disjunction
  - Intersection ∩ - represents conjunction
  - Composition • - represents transitivity
- Consistency can be checked using path consistency

Allen's interval algebra

- Represents time as intervals and relations between them
- There are 13 simple relations
- From simple relations can be composed 2^13 relations
- Operations are ∩, ∪ and •
- Checking consistency for interval algebra is a NP-complete problem
- Path consistency algorithm is not complete method for checking consistency

Conceptual neighborhood





Conceptual neighborhood of relations between two intervals

Conceptual Neighborhood

- Coarse relations
- Conceptual hierarchies can represent incomplete knowledge
- Efficient non-disjunctive reasoning
- Natural correspondence to everyday concepts
- Reduces computational complexity to polynomial time
- Can be defined at arbitrary granularity

Conceptual neighborhood

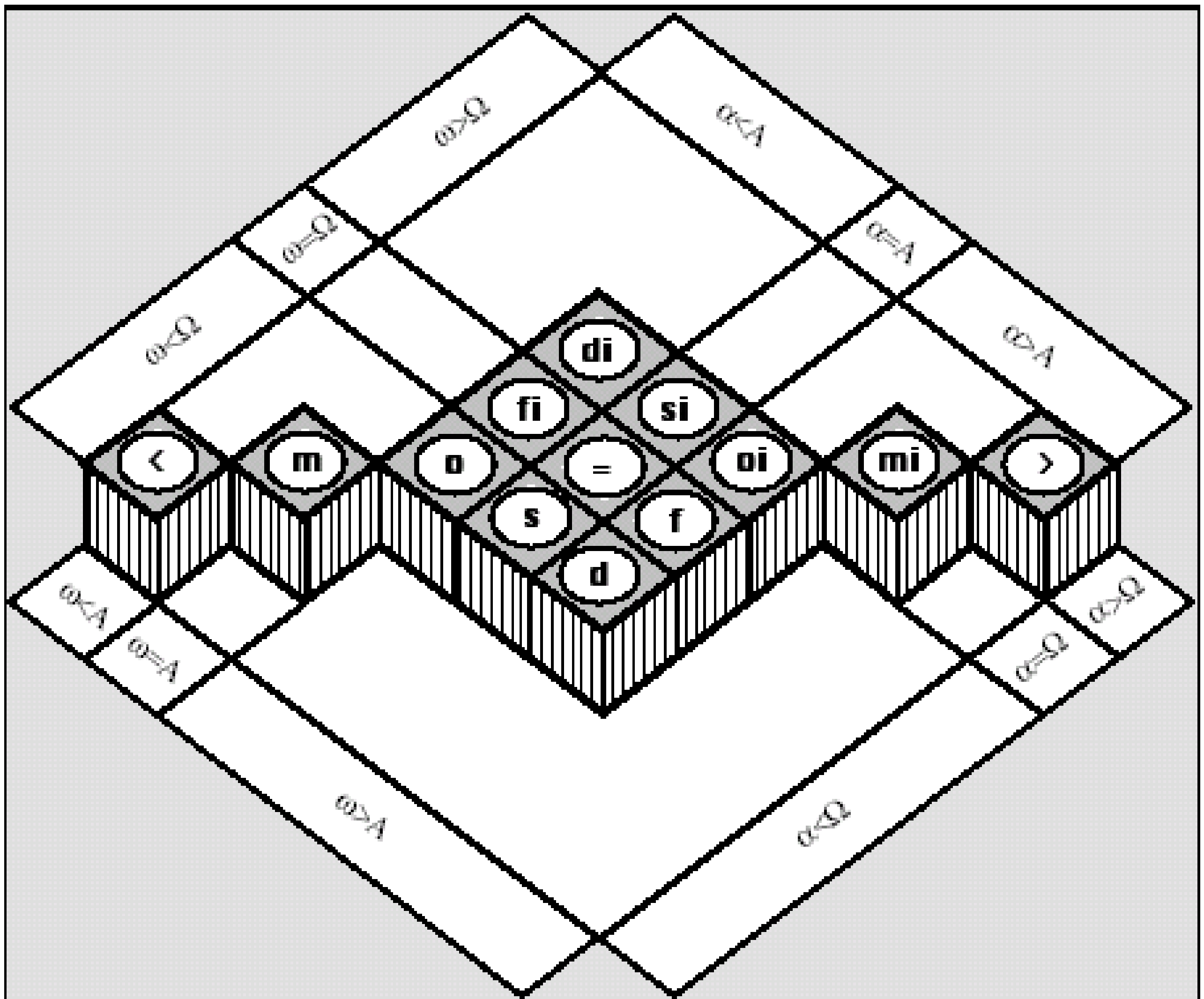
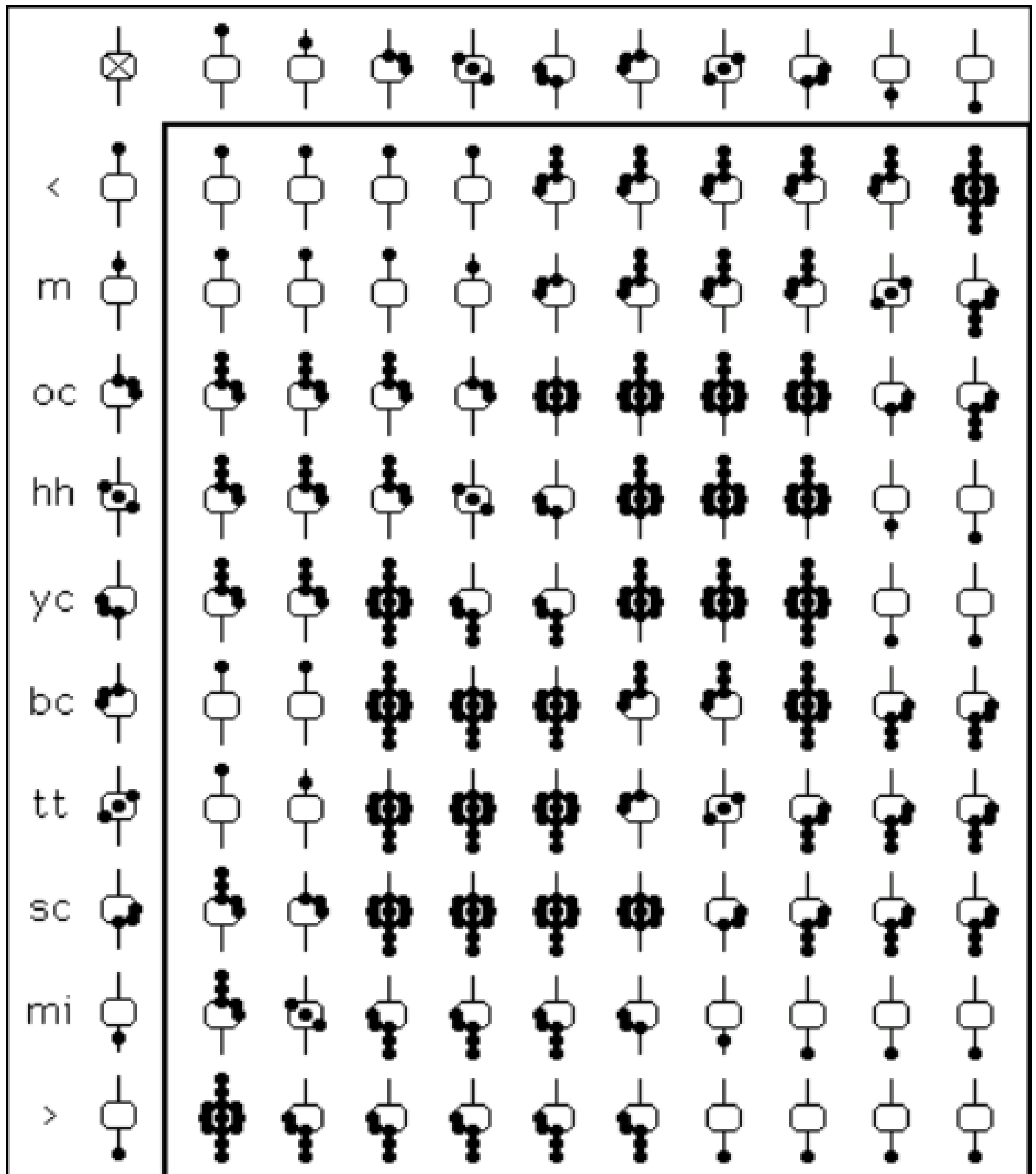


Figure 1: The thirteen qualitative relations between two events characterized by relations between their beginnings α, A and their endings ω, Ω.

Composition table



Qualitative spatial reasoning

- Reasoning about spatially distributed data and their relationships qualitatively
- Much more complex than time
- Multidimensionality - higher degree of freedom
- We can model many different aspects of space
- The most important aspects of space are topology, orientation, and distance
- Common expressions in natural language for different aspects:
  - Direction: left, above
  - Distance: far, near
  - Size: large, tiny
  - Shape: oval, convex
  - Topology: touch, inside

Qualitative spatial reasoning

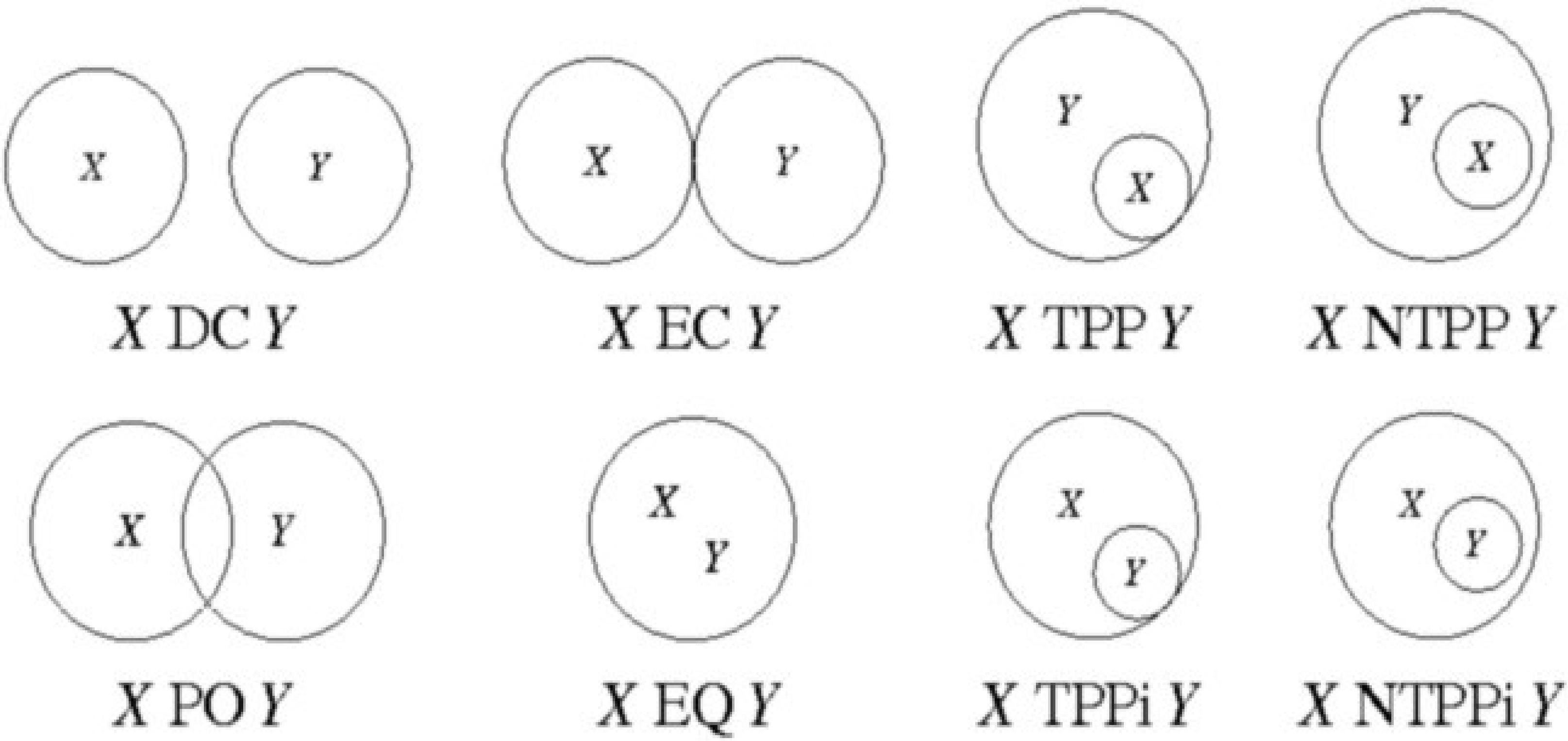
- Basic entities: points, spatial regions
- Reasoning methods include constraint based techniques used also in temporal reasoning

Region connection calculi (RCC-8)

- Describes topological relations
- 8 simple relations
- Constraint language of RCC8 is tractable

RCC-8 *Relation*	*Topological Constraints*
$\langle x, y \rangle \in \mathsf{DC}$	$x \cap y = \emptyset$
$\langle x, y \rangle \in \mathsf{EC}$	$i(x) \cap i(y) = \emptyset,\ x \cap y \neq \emptyset$
$\langle x, y \rangle \in \mathsf{PO}$	$i(x) \cap i(y) \neq \emptyset,\ x \not\subseteq y,\ y \not\subseteq x$
$\langle x, y \rangle \in \mathsf{TPP}$	$x \subset y,\ x \not\subseteq i(y)$
$\langle x, y \rangle \in \mathsf{TPP}^{-1}$	$y \subset x,\ y \not\subseteq i(x)$
$\langle x, y \rangle \in \mathsf{NTPP}$	$x \subset i(y)$
$\langle x, y \rangle \in \mathsf{NTPP}^{-1}$	$y \subset i(x)$
$\langle x, y \rangle \in \mathsf{EQ}$	$x = y$

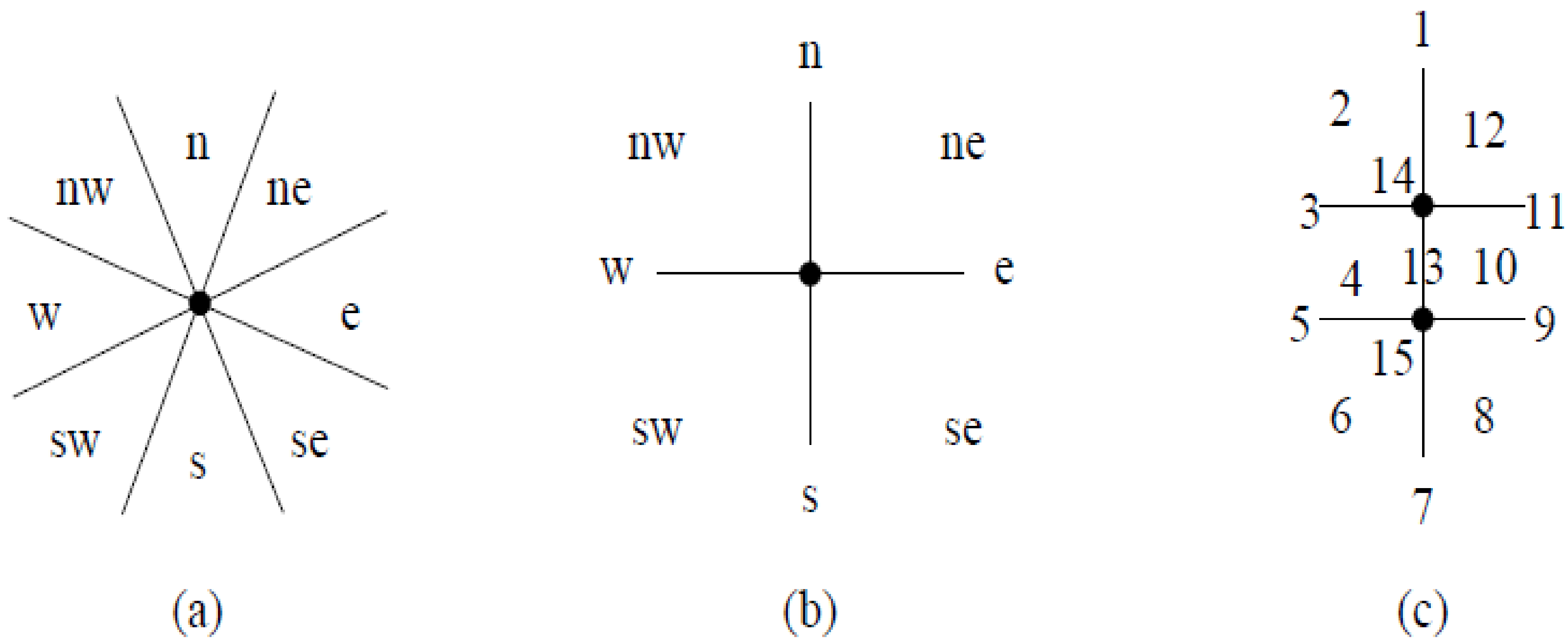
Region connection calculi (RCC-8)



Representations of orientation

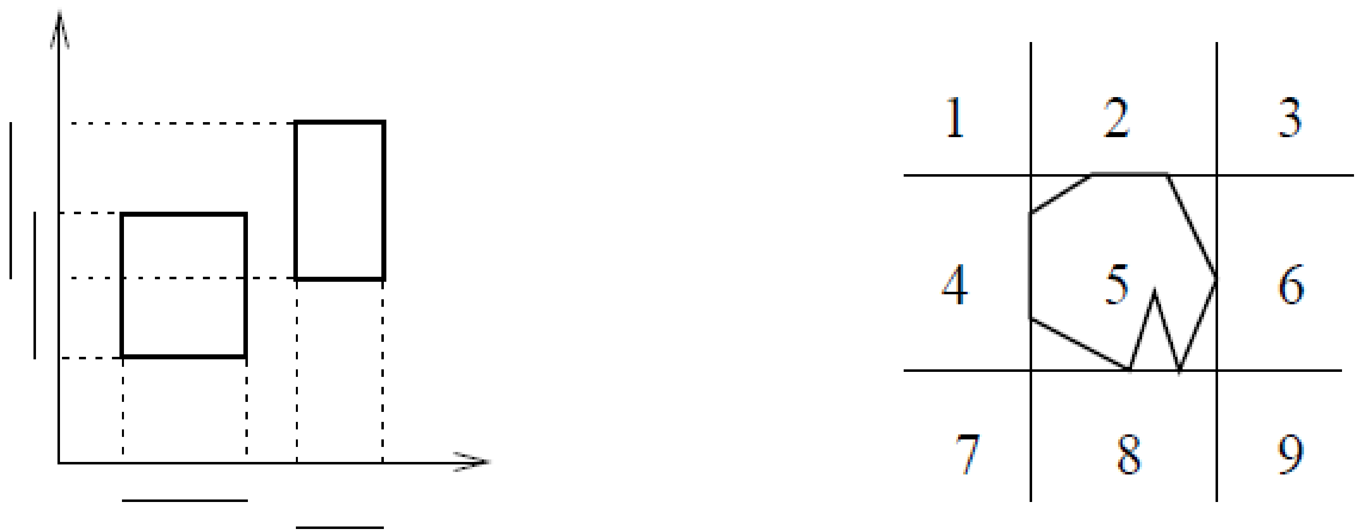
- Cone based calculus
- Projection based calculus
- Double cross calculus
- Star calculus

Representations of orientation



(a) Cone-based
(b) Projection-based
(c) Double-cross

Orientation relations between extended entities

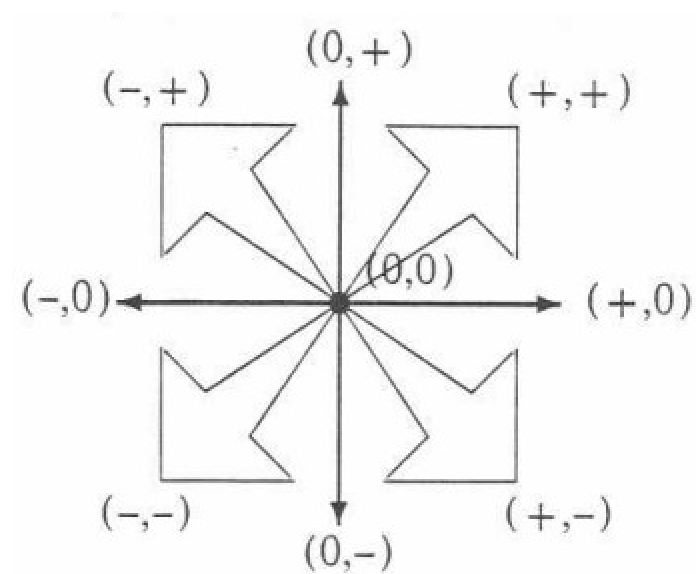


(a) (b)

(a) Rectangle algebra
(b) Direction-relation matrix

Qualitative vectors

- Vectors can represented as an ordered list, where all members are from the same domain of qualitative values
- Common case is to use domain {-, 0, +}
- Another approach is to describe vectors with qualitatively represented magnitude and direction



Qualitative **trajectory** calculus (QTC)

- Van de Weghe 2004
- Reasoning about moving objects
- Relative motion between points
- Conceptual neighborhood and composition tables
- Used in GIS tools and psychology studies

Other known calculi

- Ligozat's flip-flop calculus
- Combination of interval algebra with RCC-8
- Spatio-temporal constraint calculus

SparQ - Spatial Reasoning done Qualitatively

- A toolbox for qualitative spatial reasoning in applications
- Reference implementations for spatial calculi
- Integrable into own applications
- Free software under GNU GPL
- Linux, Mac OS X, or Solaris

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