Artificial Intelligence 1

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Constraint Satisfaction

So far we assumed the world states as blackboxes (no internal structure was assumed) accessed via:

- successor function
- goal test
- heuristic function (distance to goal)

Today we will look inside the states:

- representing problems as **constraint satisfaction problems (CSPs)**
  - state has a structure that can be exploited during problem solving
- general **constraint satisfaction techniques**
  - depth-first search combined with inference via constraint propagation

Sudoku?

Logic-based puzzle, whose goal is to enter digits 1-9 in cells of 9×9 table in such a way, that no digit appears twice or more in every row, column, and 3×3 sub-grid.

9	6	3	1	7	4	2	5	8
1	7	8	3	2	5	6	4	9
2	5	4	6	8	9	7	3	1
8	2	1	4	3	7	5	9	6
4	9	6	8	5	2	3	1	7
7	3	5	9	6	1	8	2	4
5	8	9	7	1	3	4	6	2
3	1	7	2	4	6	9	8	5
6	4	2	5	9	8	1	7	3

A bit of history

1979: first published in New York

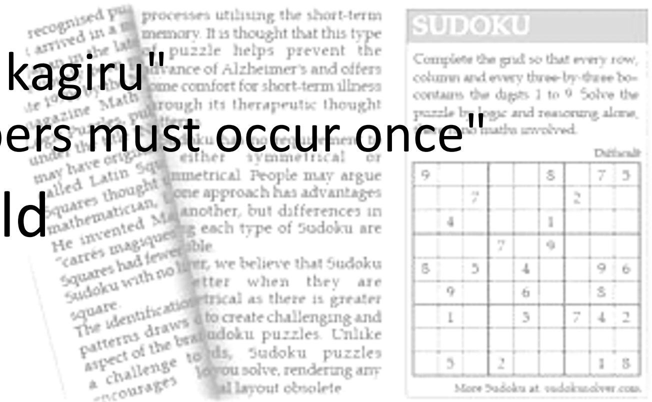
under the name „Number Place“

1986: became popular in Japan

Sudoku – from Japanes "**Su**dji wa **doku**shin ni kagiru"

"the numbers must be single" or "the numbers must occur once"

2005: became popular in the western world



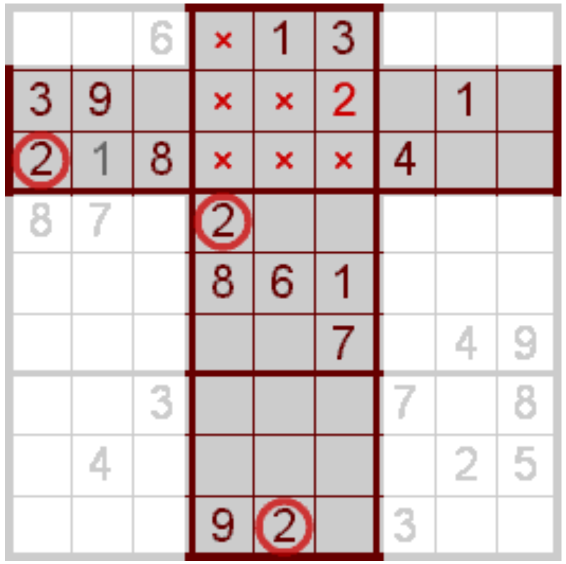
How to find out which digit to fill in?

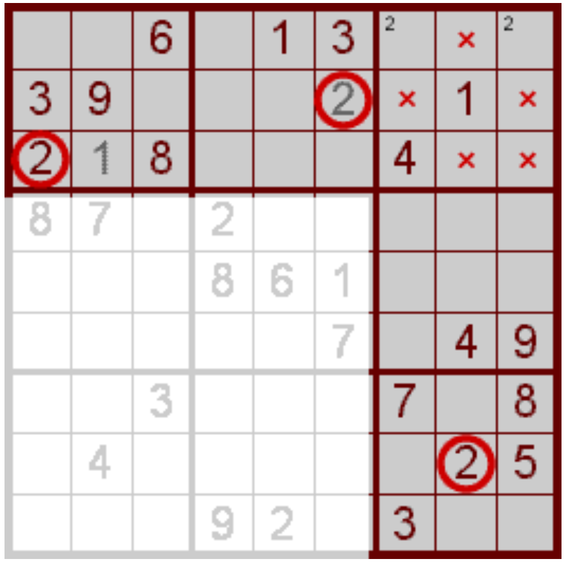
- Use information that each digit appears exactly once in each row and column.

What if this is not enough?

- Look at columns

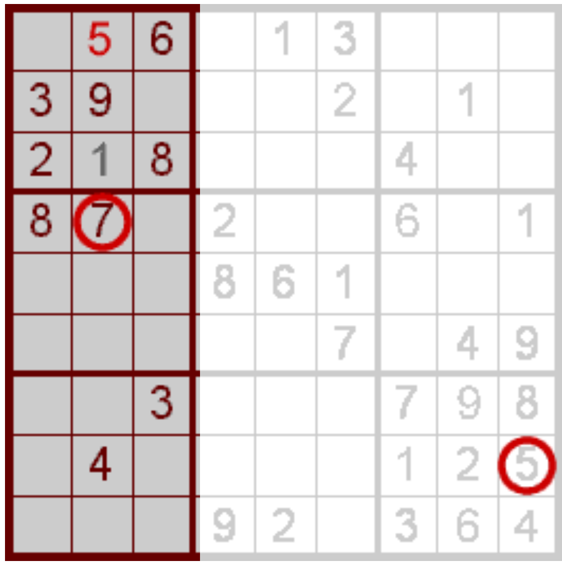
  or combine information from rows and columns

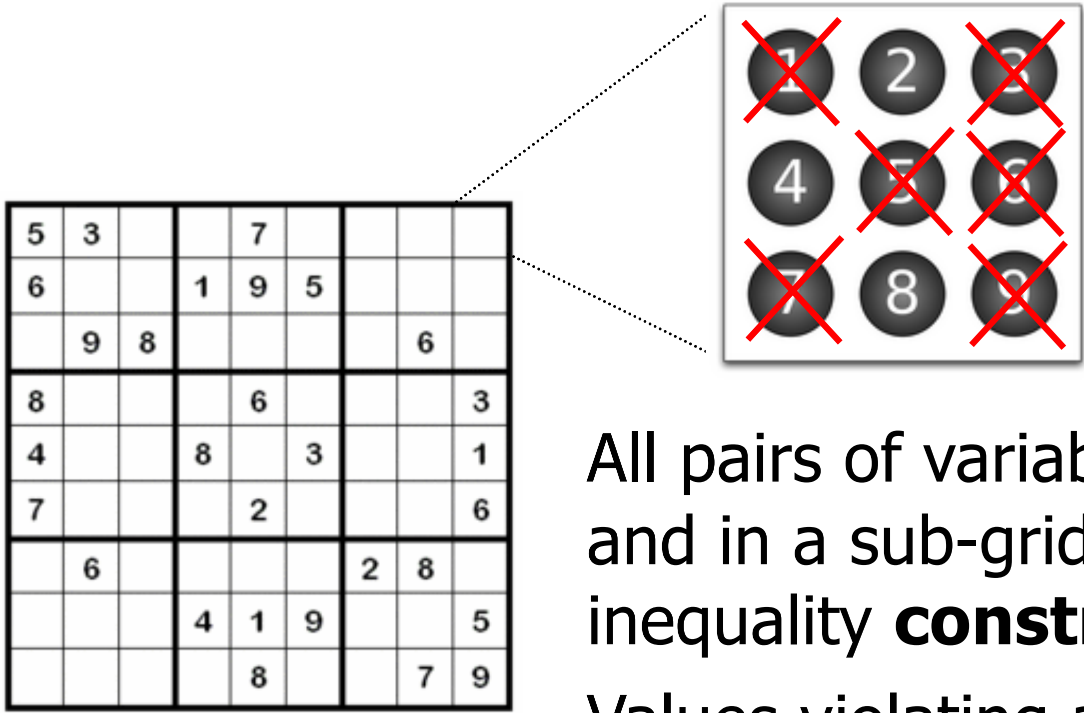




If neither rows nor columns provide enough information, we can note allowed digits in each cell.

The position of a digit can be inferred from positions of other digits and restrictions of Sudoku that each digit appears once in a column (row, sub-grid).





Each cell can be represented as a **variable** with values taken from a **domain** {1,…,9}.

All pairs of variables in a row, in a column, and in a sub-grid are connected by inequality **constraints**.

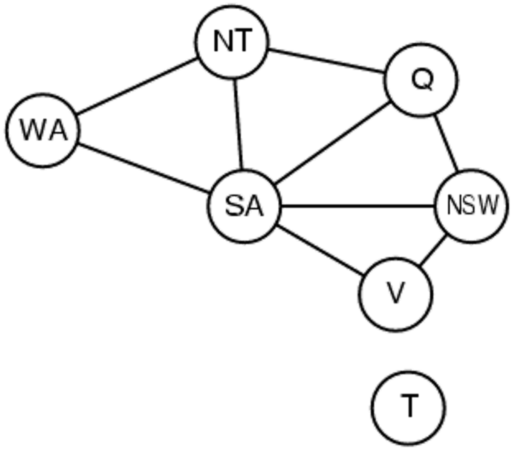
Values violating any constraint are **filtered out**.

Such a formulation of problem is called a **constraint satisfaction problem.**

Constraint satisfaction problem consists of:

- a finite set of **variables**
  - describe some features of the world state that we are looking for, for example position of queens at a chessboard
- **domains** – finite sets of values for each variable
  - describe “options” that are available, for example the rows for queens
  - sometimes, there is a single common “superdomain” and domains for particular variables are defined via unary constraints
- a finite set of **constraints**
  - a constraint is a relation over a subset of variables for example rowA $\neq$ rowB
  - a constraint can be defined in extension (a set of tuples satisfying the constraint) or using a formula (see above)

Find colours for states (red, blue, green) such that no neighbours are coloured by the same colour.



Constraint model

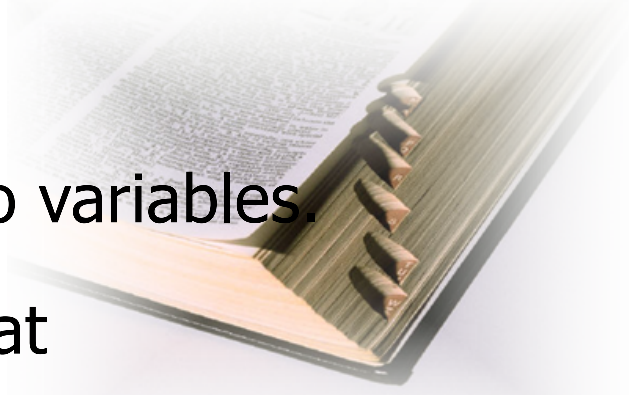
- variables: {WA, NT, Q, NSW, V, SA, T}
- superdomain: {r, b, g}
- constraints: WA ≠ NT, WA ≠ SA ...

Can also be represented as a **constraint network** (nodes = variables, arc = constraints)

Problem solution

WA = r, NT = g, Q = r, NSW = g,
V = r, SA = b, T = g

Terminology



State is a partial assignment of values to variables.

A consistent state is an assignment that does not violate any constraint.

A complete state is a state where each variable is assigned to some value.

The goal is a complete consistent state.

Sometimes, there is an **objective function** defined over the variables that evaluates the goal states by assigning them real numbers.

Then we are looking for an **optimal goal state**, that is, a goal state with the minimal (or maximal) value of the objective function.

So far we know various **search algorithms**, so we can apply them to CSPs too.

- **the initial state**: an empty assignment
- **applicable actions**: assigning a value to a certain variable such that no constraint is violated
- **the goal**: a complete consistent assignment

Some notes:

- the same solving approach for all CSPs
- the goal state is always at depth **n**, where **n** is the number of variables
  - We can use DFS even without checking cycles!
- the order of actions is not important to reach the goal (a CSP is a **commutative problem**)
  - ⟨WA=r, NT=g⟩ is the same as ⟨NT=g, WA=r⟩
  - we can also use local search techniques
- it is possible to use different branching schemes to solve CSPs, for example domain splitting

The core uninformed algorithm to solve a CSP:

- gradually assigns values to variables
- if no value can be assigned to a variable then goes back to the previous variable and tries an alternative value for that variable

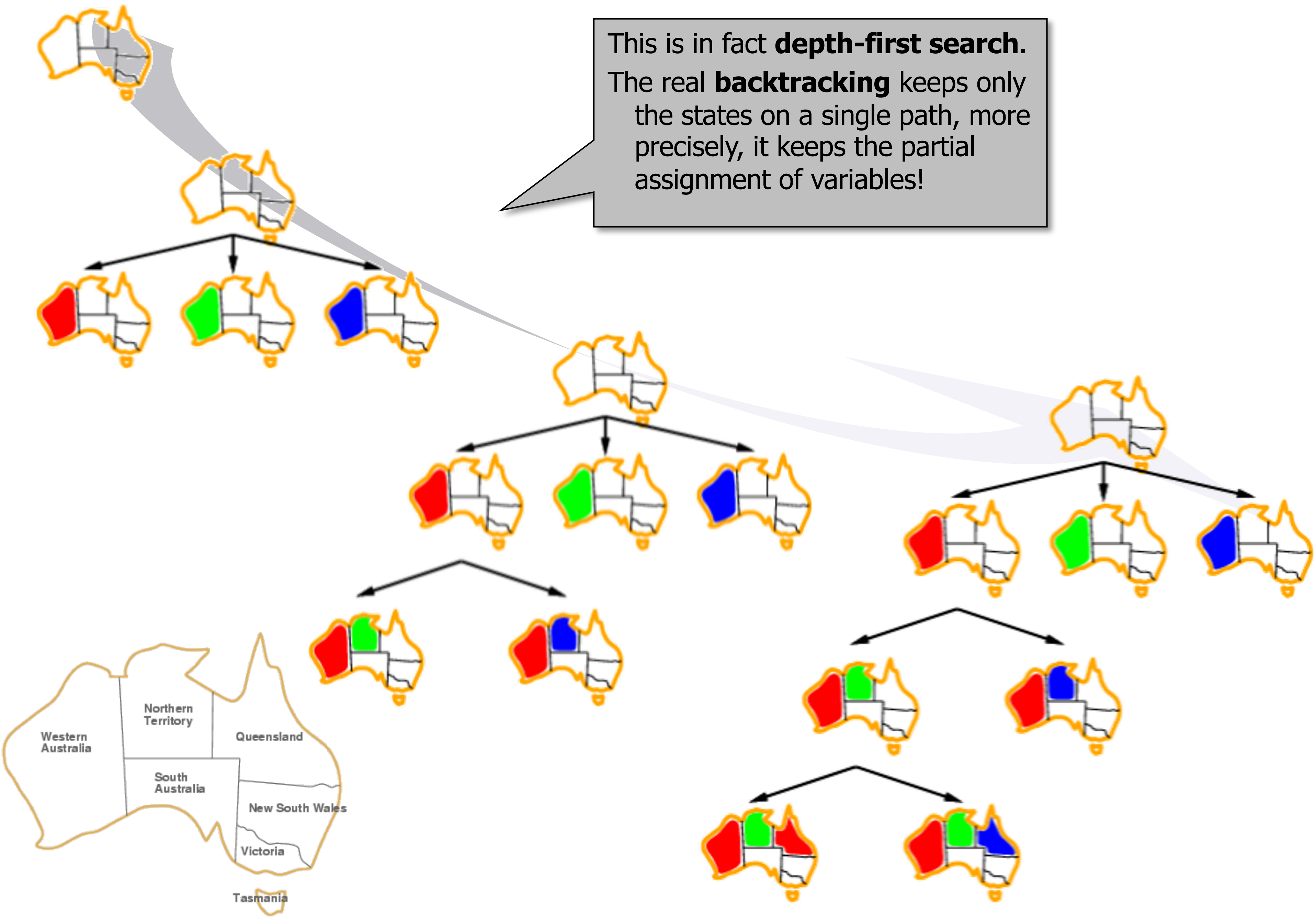
```
function BACKTRACKING-SEARCH(csp) returns a solution or failure
  return BACKTRACK(csp, { })

function BACKTRACK(csp, assignment) returns a solution or failure
  if assignment is complete then return assignment
  var ← SELECT-UNASSIGNED-VARIABLE(csp, assignment)
  for each value in ORDER-DOMAIN-VALUES(csp, var, assignment) do
     if value is consistent with assignment then
        add {var = value} to assignment
        inferences ← INFERENCE(csp, var, assignment)
        if inferences ≠ failure then
           add inferences to csp
           result ← BACKTRACK(csp, assignment)
           if result ≠ failure then return result
           remove inferences from csp
        remove {var = value} from assignment
  return failure
```

Backtracking: an example

This is in fact **depth-first search**.

The real **backtracking** keeps only the states on a single path, more precisely, it keeps the partial assignment of variables!

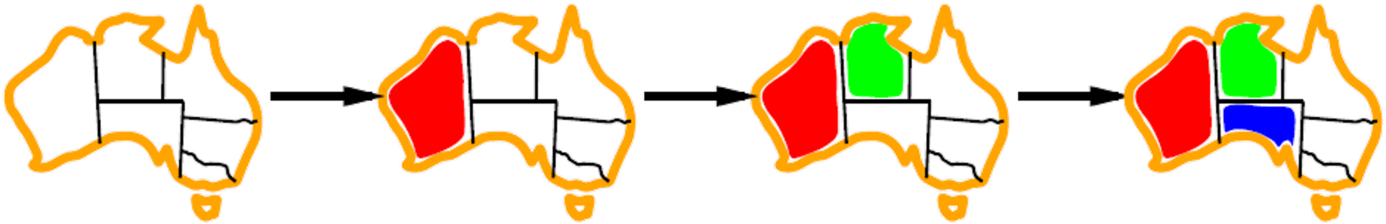


How to influence efficiency of search?

- **the choice of variable for assignment**
  - at the end, we need to assign values to all the variables, but the order of variables influences the size of the search tree
  - problem independent heuristics (such as fail-first)
- **assigning the "right" values**
  - this is usually problem dependent
- **early detection of "wrong" branches**
  - deducing extra information (for example via constraint propagation)
- **exploiting a problem structure**
  - some problems can be solved using backtrack-free search (for example tree-structured CSPs)

Backtracking: variable ordering

- **The most restricted** variable first
  - a variable with the smallest number of actions
  - i.e. variable with the smallest current domain
  - so called **dom heuristic**



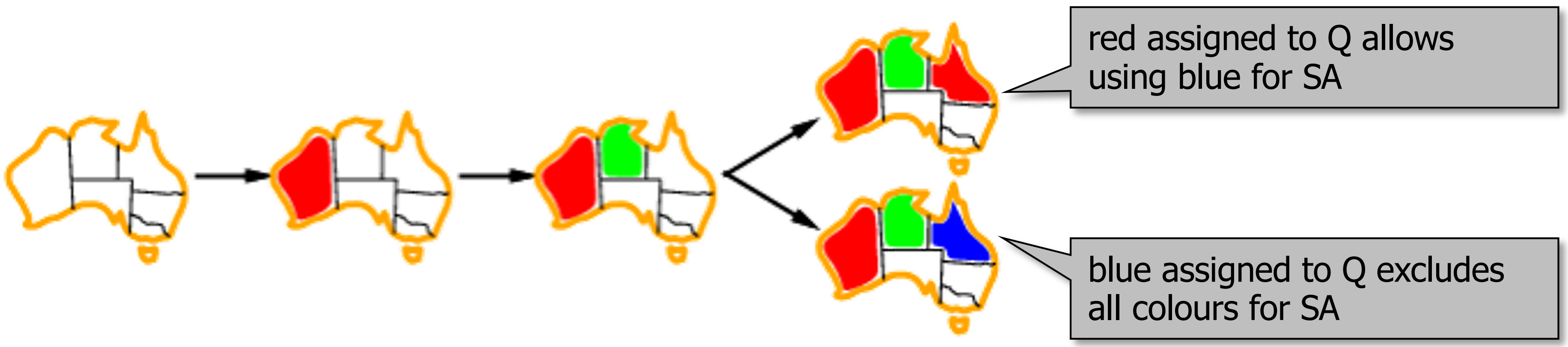
- **The most constrained** variable first
  - participates in the largest number of constraints
  - so called **deg heuristic**
  - frequently used when dom heuristic does not select a single variable (**dom+deg heuristic**)

These are instances of the **fail-first principle** – assign first a variable whose assignment will probably lead to a failure.

When selecting a value for the variable, we prefer values belonging to a solution with a high chance – a **succeed-first principle**.

How to recognize such a value?

– for example a value that **restricts least the other variables** (keeps the largest flexibility in the problem)

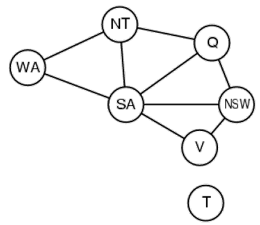


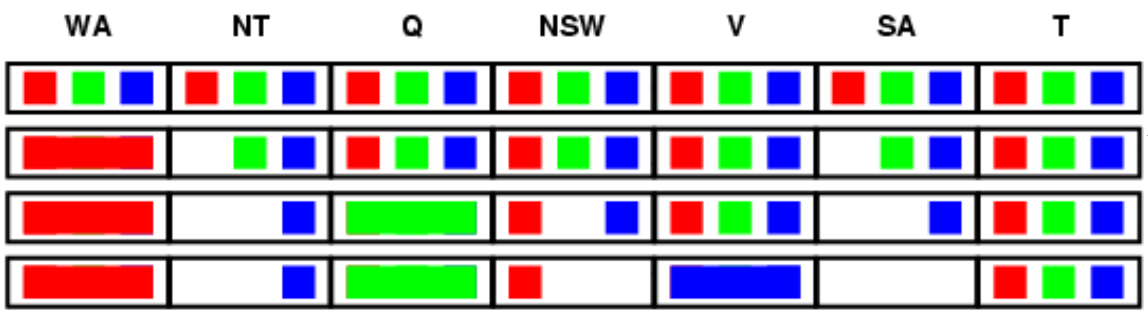
– the value can also be found by relaxing the problem, finding the solution of the relaxed problem, and using values from this solution (recall construction of heuristics)

– finding the generally best value is frequently computationally expensive and hence **problem-dependent heuristics** are usually preferred

Can we guess in advance that a given path does not lead to the goal?

- After assigning a value to the variable we can check the future constraints – constraints between the current variable and not-yet instantiated variables – **forward checking.**
- constraint check = remove values violating the constraint



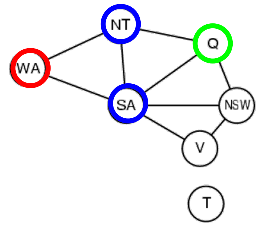


after assigning red to WA we can remove that colour from NT and SA

after assigning green to Q we can remove green from NT, SA, NSW

after assigning blue to V we remove blue from NSW and SA and the **domain** for SA becomes **empty** so we backtrack immediately

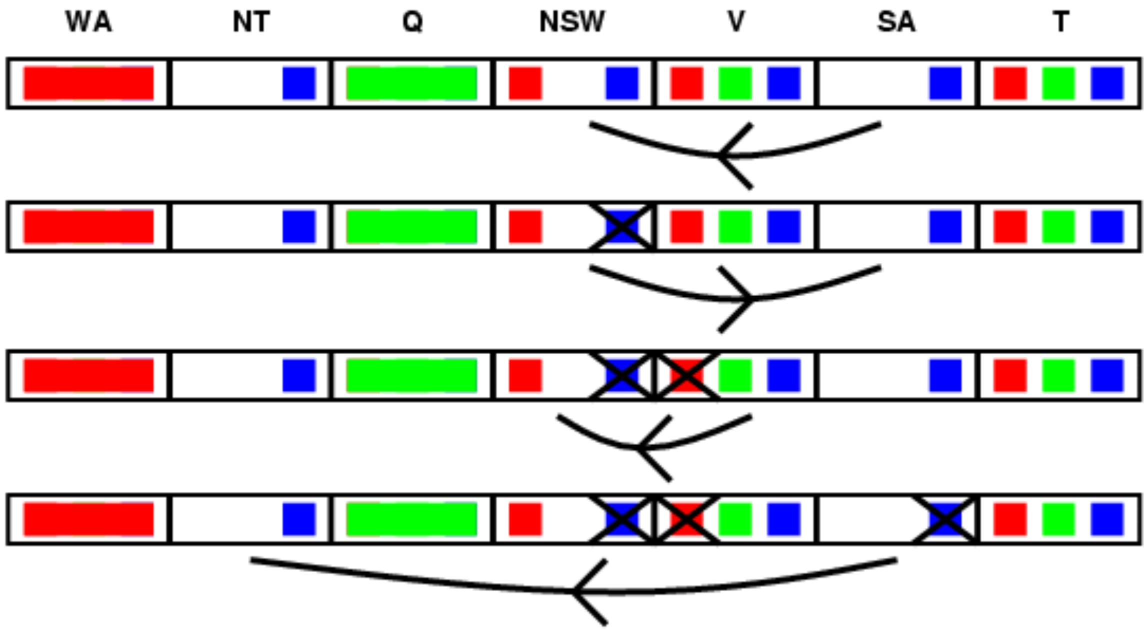
Can we exploit the constraints even more?



- we can check the constraints even between the future variables; then we can find that blue cannot be used for NT and SA and this is the only colour in their domains
- because the assigned value is propagated through the constraints, this method is called **constraint propagation** or **look ahead**
- this is implemented via maintaining **consistency of constraints**

Arc consistency

- each constraint is used to filter out values that violate the constraint
- usually implemented in a directional way – remove values from the domain of X that have no support (a consistent value) in the domain of Y for the binary constraint (X,Y); of course do it also in the reverse direction



– domain filtering in X is done each time the domain of Y changes

– filtering is repeated while the domains are changing until reaching a fixed point or emptying some domain

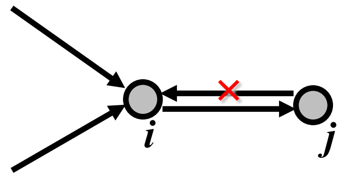
The algorithm can be applied incrementally during search – when X is instantiated put all constraints related to X to the queue.

function AC-3(*csp*) **returns** false if an inconsistency is found and true otherwise
  queue ← a queue of arcs, initially all the arcs in *csp*

  while *queue* is not empty **do**
    (X_i, X_j) ← POP(*queue*)
    if REVISE(*csp*, X_i, X_j) **then**
      if size of $D_i = 0$ **then return** *false*
      for each X_k **in** X_i.NEIGHBORS - $\{X_j\}$ **do**
        add (X_k, X_i) to *queue*
  return *true*

If the domain of variable X_i changed then verify all arcs (constraints) leading to the variable except the arc from the variable X_j.

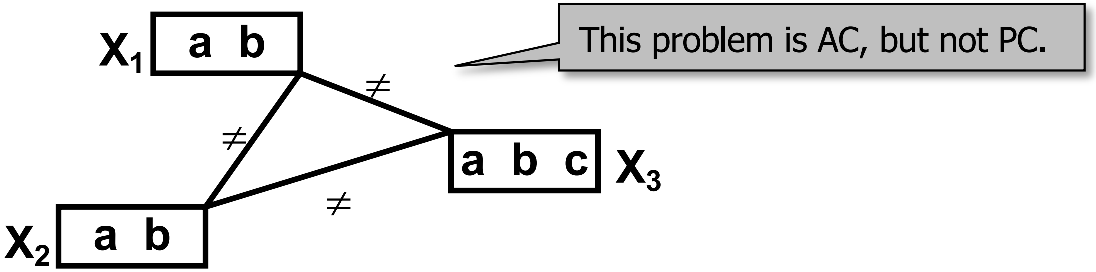
function REVISE(*csp*, X_i, X_j) **returns** true iff we revise the domain of X_i
  revised ← *false*
  for each x **in** D_i **do**
    if no value y in D_j allows (x,y) to satisfy the constraint between X_i and X_j **then**
      delete x from D_i
      revised ← *true*
  return *revised*



Domain filtering for variable X_i removes values that have no support in the variable X_j, also, if any value is deleted this information is passed to the calling procedure. Knowing constraint semantics can speedup constraint checking (for example X<Y).

Time complexity of AC-3 is $O(ed^3)$, where e is the number of constraints and d is the size of domain – we need to repeatedly (ed) check the constraints (d^2). This is not optimal, we can remember the result of consistency checks - AC-4, AC-3.1, AC-2001 with time complexity $O(ed^2)$.

- We can generally define **k-consistency**, as the consistency check where for a consistent assignment of (k-1) variables we require a consistent value in one more given variable.
  - arc consistency (AC) = 2-consistency
  - path consistency (PC) = 3-consistency



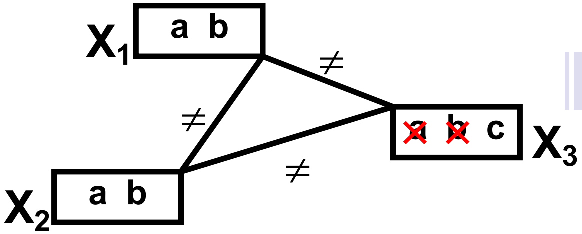
- If the problem is i-consistent $\forall i=1,..,n$ (n is the number of variables), then we can solve it in a backtrack-free way.
  - DFS can always find a value consistent with the assignment of previous variables
- Unfortunately, the time complexity of k-consistency is exponential in k.

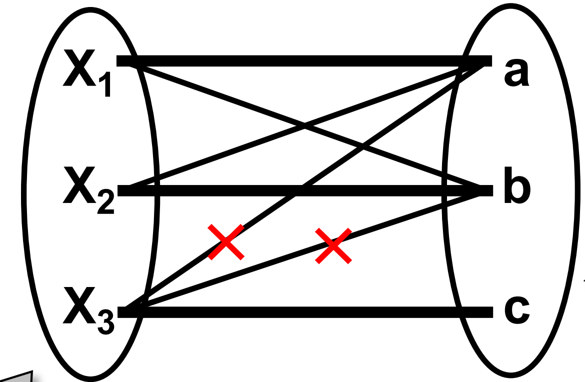
Instead of stronger consistency techniques (expensive) usually **global constraints** are used – a global constraint encapsulates a sub-problem with a specific structure that can be exploited in the ad-hoc domain filtering procedure.

Example:

global constraint **all_different({X_1,…, X_k})**

- encapsulates a set of binary inequalities $X_1 \neq X_2$, $X_1 \neq X_3$, …, $X_{k-1} \neq X_k$
- **all_different({X_1,…, X_k})** = $\{(d_1,\ldots, d_k) \mid \forall i\ \ d_i \in D_i\ \ \&\ \forall i \neq j\ \ d_i \neq d_j\}$
- the filtering procedure is based on matching in bipartite graphs





1. find a **maximal matching**
2. **remove arcs** that are not part of any maximal matching
3. **remove** corresponding **values**

Bipartite graph

- variables on one side, values on the other side
- arcs connect a variable with values in its domain

A declarative approach to problem solving

- construct a **model** (variables, domains, constraints)
- use a **general constraint solver**

Possible extensions

- **optimisation problems**
  - applying branch-and-bound
- **soft constraints**
  - constraints describe preferences rather than restrictions
  - optimisation methods are applied there

Other solving approaches

- **local search** (the path to the goal is not important)
- integer programming (for linear constraints)

Bioinformatics

- DNA sequencing
- determining 3D structures of proteins

Planning

- autonomous action planning for space probes (Deep Space 1)

Manufacturing scheduling

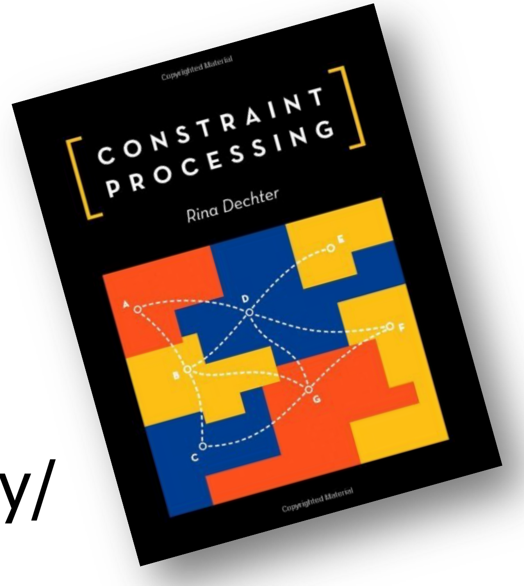
- savings after applying CSP: US$ 0.2-1 milion per day

Constraint Solvers

- SICStus Prolog (available in labs)
- SWI Prolog
- ECLiPSe (Open Source, http://eclipse.crosscoreop.com/)
- GECODE (Open Source C++, http://www.gecode.org/)
- Choco (Open Source Java, http://www.emn.fr/z-info/choco-solver/)
- CP Optimizer
- …

Course **Constraint Programming**

- also taught in English
- Winter term
- http://ktiml.mff.cuni.cz/~bartak/podminky/



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