Planning & Scheduling

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Temporal and Resource Planning

Temporal planning

Temporal planning involves reasoning on time.

Actions do not describe state transitions only but they specify how the **state variables evolve in time** and what are the **prevailing conditions**:

- actions have **duration**
  - going from A to B takes some time
- **preconditions** must hold at specific time of action execution
  - place B must be free right before arrival
- similarly action **effects** happen at specific times of the action
  - place A is made empty right after leaving it
- actions can **interfere** to achieve a **joint effect**
  - to open doors we need to press the handle and push (or pull) the doors
- **goals** and **known intermediate states** can be spread in time
  - a dock is closed for a given time interval due to maintenance so vessels cannot use it
  - customer A will be served before the customer B

Approaches to temporal planning

- **Planning with temporal operators**
  - Action specification contains information when the preconditions must hold, when the effects become active and there are temporal relations between the time points and intervals.
- **Planning with chronicles**
  - Actions describe partially defined functions how the state variables are being changed in time.
- **Planning graph and time**
  - Actions are split into three parts – start, middle, and end – and state layers have duration.

State variables and time

- Multi-valued state variables describe some properties depending on world states.
  - rloc: robots × S → locations
- Now **state variables** will depend on exact **time**:
  - rloc: robots × time → locations

Example:

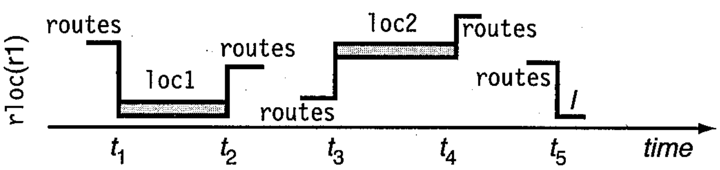
- At time t_1 robot r1 entered place loc1, where it stayed till time t_2 and then left.
- At time t_3, t_2<t_3, robot r1 arrived to place loc2, where it stayed till time t_4 and then left.
- At time t_5, t_4<t_5, robot r1 arrived to some not-yet specified place l.

- The evolution of a state variable can be specified partially with ”holes” where the value is unknown.
  - During planning, this evolution will be concretised.
- We will restrict to **piecewise constant functions** that can be described using two types of **temporal assertions**:
  - **event $x@t:(v_1,v_2)$** specifies the instantaneous change of the value of x from v_1 to v_2 ($v_1 \neq v_2$) at time t
    - $x@t:(v_1,v_2) \equiv (\exists t_0\ \forall t'\ (t_0 < t' < t)\ x(t')=v_1) \wedge\ x(t)=v_2$
  - **persistence condition $x@[t_1,t_2):u$** specifies that the value of x persists as being equal to u over the interval $[t_1,t_2)$
    - $x@[t_1,t_2):u \equiv \forall t\ (t_1 \leq t < t_2)\ x(t)=u$

There is the following relation between events and persistence conditions:

$x@t:(v_1,v_2) \equiv\ v_1 \neq v_2 \wedge \exists t_1, t_2\ (t_1<t<t_2)\ x@[t_1,t):v_1 \wedge x@[t,t_2):v_2$

- A **chronicle** for a set of state variables is a pair Φ=(F,C), where:
  - F is a set of **temporal assertions** over the state variables (i.e. events and persistence conditions)
  - C is a set of constraints of two types:
    - **object constraints**, i.e., constraints connecting object variables in the form of $x \in D$, $x=y$, $x \neq y$ and rigid relations
    - **temporal constraints**, i.e., constraints over the temporal variables using the point algebra (<,=,>)
- **Timeline** is a chronicle for a single state variable.



({ $rloc(r1)@t_1: (l_1,loc1)$,
$rloc(r1)@[t_1,t_2) : loc1$,
$rloc(r1)@t_2: (loc1,l_2)$,
$rloc(r1)@t_3: (l_3,loc2)$,
$rloc(r1)@[t_3,t_4) : loc2$,
$rloc(r1)@t_4: (loc2,l_4)$,
$rloc(r1)@t_5: (l_5,l)$ }

{ $adjacent(l_1,loc1)$,
$adjacent(loc1,l_2)$,
$adjacent(l_3,loc2)$,
$adjacent(loc2,l_4)$,
$adjacent(l_5,l)$,
$t_1<t_2<t_3<t_4<t_5$ })

Separation constraints

- To ensure that the **timeline can specify a valid evolution** of a state variable, there must **not be any two conflicting temporal assertions** – temporal assertions that allow different values of the state variable at the same time.
- Temporal conflicts can be avoided by requiring a timeline to contain, either explicitly or implicitly, separation constraints that make each pair of assertions non-conflicting.
- The **separation constraint** for a pair assertions is defined as follows:
  – for $x@[t_1,t_2):v_1$ a $x@[t_3,t_4):v_2$ there are three possible separation constraints:
    - $t_2 \leq t_3$, $t_4 \leq t_1$, $v_1 = v_2$
  – for $x@t:(v_1,v_2)$ a $x@[t_1,t_2):v$ there are four possible separation constraints:
    - $t < t_1$, $t_2 < t$, $(t_1 = t \wedge v = v_2)$, $(t_2 = t \wedge v = v_1)$
  – for $x@t:(v_1,v_2)$ a $x@t':(v_1',v_2')$ there are two possible separation constraints:
    - $t \neq t'$, $(v_1 = v_1' \wedge v_2 = v_2')$

Note:

– Assertions can also be separated by constraints on difference of the object variables in the assertions (or example assertions for state variables rloc(r) and rloc(r') can be separated by a constraint $r \neq r'$).

Consistency

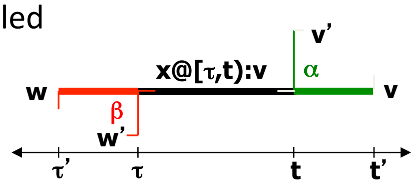
- **Timeline** Φ=(F,C) for the state variable x is **consistent** iff C is consistent (there is a solution) and for each pair of temporal assertions from F there is a separation constraint entailed by C.
  – the separation constraint can be a part of C
  – or it can be entailed by C (to be true in any solution of C)
- A **chronicle** is **consistent** iff all its timelines are consistent.

Note:

– Consistency requires the separation constraints to be entailed by C; it is not enough if the separation constraints can be added to C without a conflict.

Support for assertion

- A consistent **chronicle** Φ=(F,C) **supports an assertion** α (α being either **x@t:(v,v')** or **x@[t,t'):v**) iff there is in F an assertion β that asserts a value w for α (β is either **x@τ:(w',w)** or **x@[τ',τ):w**) and there exists a set of separation constraints c such that $\Phi \cup (\{\alpha, x@[\tau,t):v\}, \{w=v, \tau<t\} \cup c)$ is a consistent chronicle.
  - $\Phi \cup \Phi' = (F \cup F', C \cup C')$, $\Phi \subseteq \Phi' \equiv (F \subseteq F' \wedge C \subseteq C')$,
  - β is called a **support** for α in α
  - the pair δ = **($\{\alpha$, x@[τ,t):v$\}$, $\{$w=v,τ<t$\}$**$\cup$**c)** is called an **enabler** for α in Φ



- **Notes:**
  - The chronicle must be consistent before enabling α.
  - The enabler is a chronicle.
  - The support for α is looked only for value v, that is before the time t. This is because the support will be used as a causal explanation for α.
  - There can be several ways to enable an assertion α in Φ.

Support for chronicle

A consistent **chronicle** Φ=(F,C) **supports a set of assertions** ε iff each assertion $\alpha_i \in \varepsilon$ is supported by $(F \cup \varepsilon - \{\alpha_i\}, C)$ with an enabler δ_i such that $\Phi \cup \phi$ is a consistent chronicle, where $\phi = \cup_i \delta_i$.

Notes:

- The definition allows an assertion $\alpha_i \in \varepsilon$ to support another assertion $\alpha_j \in \varepsilon$ with respect to Φ as long as the union of the enablers is consistent with Φ. This allows synchronisation of several actions with **interfering effects**.
- ϕ is called an **enabler** for ε (again, the enabler is not unique)

Let Φ'=(F',C') be a chronicle such that Φ supports F' and let **$\theta(\Phi'/\Phi)$ = $\{\phi \cup (\emptyset$,C'$)$ | ϕ is enabler for F'$\}$** be a set of all possible enablers. Then a consistent **chronicle** Φ=(F,C) **supports chronicle** Φ'=(F',C'), iff Φ supports F' and there is an enabler $\phi \in \theta(\Phi'/\Phi)$ such that $\Phi \cup \phi$ is consistent chronicle.

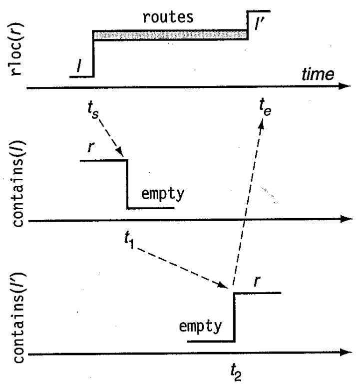
Φ **entails** Φ' iff Φ supports Φ' and there is an enabler $\phi \in \theta(\Phi'/\Phi)$ such that $\phi \subseteq \Phi$.

A chronicle planning operator is a pair o = (name(o), (F(o),C(o))):

- name(o) is a syntactic expression of the form $o(t_s,t_e,t_1,\ldots,v_1,v_2,\ldots)$ containing all temporal and object variables in the operator (o is an operator symbol)
- (F(o),C(o)) is a chronicle

Example *(simplified):*

$move(t_s,t_e,t_1,t_2,r,l,l')$ =
{$rloc(r)@t_s$: (l,routes),
$rloc(r)@[t_s,t_e)$: routes,
$rloc(r)@t_e$: (routes,l'),
$contains(l)@t_1$: (r,empty),
$contains(l')@t_2$: (empty,r),
$t_s < t_1 < t_2 < t_e$,
adjacent(l,l') }



The differences from classical planning operators are

- **no distinction** between **preconditions and effects**
- **an operator is applied** not to a state but **to a chronicle**
- the result of **applying** an instance of operator to a chronicle is **not unique**

- **An action** is a partially instantiated operator.
- **Action** a=(F(a),C(a)) is **applicable** to a chronicle Φ iff Φ supports the chronicle (F(a),C(a)).

  The result of applying a to Φ is not unique but a set of chronicles $\gamma(\Phi,a) = \{\Phi \cup \phi \mid \phi \in \theta(a/\Phi)\}$.
- **A set of actions** $\pi=\{a_1,\ldots,a_n\}$ is **applicable** to Φ iff Φ supports $\Phi_\pi = \cup_i (F(a_i),C(a_i))$.

  The result of applying π to Φ is the set of chronicles $\gamma(\Phi,\pi) = \{\Phi \cup \phi \mid \phi \in \theta(\Phi_\pi/\Phi)\}$.

- A **temporal planning problem** is a triple $P=(O,\Phi_0,\Phi_g)$, where
  - O is a set of chronicle planning operators
  - Φ_0 is a consistent chronicle that represents an initial scenario describing the rigid relations, the initial state, and the expected evolution that will take place independently of the actions to be planned
  - Φ_g is a consistent chronicle that represents the goals
- A **solution plan** for a problem P is a set of actions $\pi=\{a_1,\ldots,a_n\}$, each being an instance of operator in O, such that that there is a chronicle in $\gamma(\Phi_0,\pi)$ that entails Φ_g.

- The planning procedure is derived from **plan-space planning**.
- For a planning problem $P=(O,\Phi_0,\Phi_g)$ we start with the chronicle $\Phi=(F_0,C_0\cup C_g)$, a set of open goals $G=F_g$, an empty plan $\pi=\emptyset$, and an empty set of threats $K=\emptyset$.

```
CP(Φ, G, K, π)
    if G = K = Ø then return(π)
    perform the two following steps in any order
        if G ≠ Ø then do
            select any α ∈ G
            if θ(α/Φ) ≠ Ø then return(CP(Φ, G − {α}, K ∪{θ(α/Φ)}, π))
            else do
                relevant ← {a | a contains a support for α}
                if relevant = Ø then return(failure)
                nondeterministically choose a ∈ relevant
                return(CP(Φ ∪ (F(a), C(a)), G ∪ F(a), K ∪ {θ(a/Φ)}, π ∪ {a}))
        if K ≠ Ø then do
            select any C ∈ K
            threat-resolvers ← {φ ∈ C | φ consistent with Φ}
            if threat-resolvers = Ø then return(failure)
            nondeterministically choose φ ∈ threat-resolvers
            return(CP(Φ ∪ φ, G, K − C, π))
end
```

Open goal

- is either supported by Φ, then its enablers are added to K
- otherwise, a resolver is an action that supports the goal and this action is added to the system

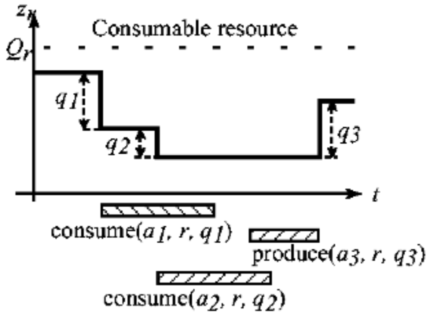
Threats is a pending set of enablers.

- From each set of enablers we need to select one that is consistent with Φ and its added to Φ.

- Now we know how to use **time in planning**
  - planning with chronicles
- We already have some **resources** in planning
  - for example a hand or a crane
- A **state variable** with two values occupied/empty is not an efficient model to describe several identical resources – it does not matter which hand is used to pick up the block (the hands are symmetrical).
- We can model a set of identical unary resources using a **single multi-valued state variable** describing the **number of available resources**.
  - the domain for the variable is **numeric** (the number of resources)
  - changes of values are **relative** (the resources are taken and returned)

- A state variable describes how some property of the object changes in time.
  - the changes are **absolute** (location changed from loc1 to loc2)
- Similarly we can describe the capacity profile of the resource, i.e., how the available capacity changes with time, using a **capacity variable**.
  - resources × time → {0,1,…,Q}, where Q is a maximal capacity
  - the domain is numeric
  - the changes of values are **relative** (available capacity is increased or decreased by some amount)

  Note:
  we assume **instant changes**



Temporal assertions and capacity variables

- We can describe changes of capacity variables using **temporal assertions for resources**.
  - **decrease** of capacity z@t:-q
  - **increase** of capacity z@t:+q
  - **borrowing** of capacity z@[t,t'):q

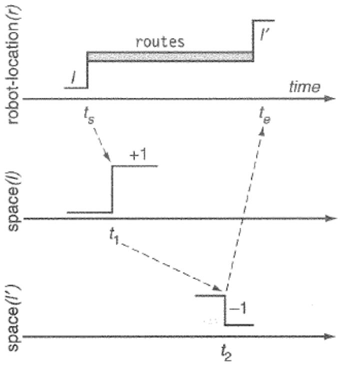
Notes:

- this is a description of **relative** changes
- z@[t,t'):q ≡ z@t:-q ∧ z@t':+q
- z@t:-q ≡ z@[t,∞):q
- z@t:+q ≡ z@0:+q ∧ z@[0,t):q
  - at the beginning we increase the capacity from Q to Q+q and we borrow the increased capacity till time t
- it is necessary to specify the maximal capacity for each capacity variable in the problem description

Operators and resources

- **Planning operator** is a chronicle with temporal assertions and constraints.
- To work with resources we need to **add** to a chronicle just the **temporal assertions for resources**.

$$\text{move}(t_s, t_e, t_1, t_2, r, l, l') = \{\ \text{robot-location}(r)@t_s : (l, \text{routes}),$$
$$\text{robot-location}(r)@[t_s, t_e) : \text{routes},$$
$$\text{robot-location}(r)@t_e : (\text{routes}, l'),$$
$$\text{space}(l)@t_1 : +1,$$
$$\text{space}(l')@t_2 : -1,$$
$$t_s < t_1 < t_2 < t_e,$$
$$\text{adjacent}(l, l')\ \}$$



Resource conflicts

- We will only assume actions that borrow resource capacity so the assertions have the form z@[t,t'):q.
- We need to extend the notion of consistency to cover assertions for resources, i.e., to assume capacity limits.
- A **set of temporal assertions** R_z for resource z is **conflicting** iff there is a subset $\{z@[t_i,t_i'):q_i \mid i\in I\}\subseteq R_z$ such that:
  - assertions from this subset overlap in time, i.e., it is possible to assign times t_i such that $\cap_{i\in I} [t_i,t_i')\neq\emptyset$
  - $\Sigma_{i\in I}\, q_i > Q$

Notes:

- Resource conflict means a possible **exceeding of resource capacity**.
- The resource conflict can only appear between the assertions for the same resource variable.

- A **chronicle is consistent** iff all temporal assertions over all state variables are consistent and there is no conflicting set of assertions for capacity variables.

Critical set

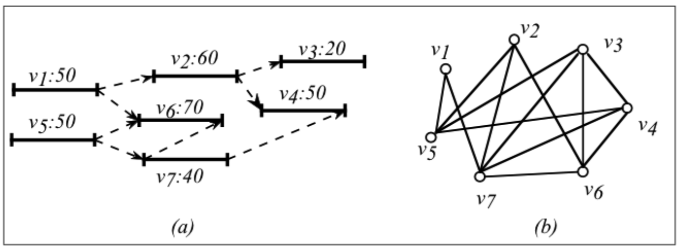
How to discover resource conflicts?

Claim:

Intervals from a set I can overlap iff any pair of intervals from I can overlap.
$(\cap_{i\in I} [t_i,t_i')\neq\emptyset \Leftrightarrow \forall i,j\in I: [t_i,t_i')\cap[t_j,t_j')\neq\emptyset)$

The set of intervals/assertions can be represented using a **graph**:

- nodes describe intervals/assertions
- edges connect nodes with overlapping intervals



- We will look for a clique U in the graph such that $\Sigma_{i\in U}\, q_i > Q$. More precisely, we will look for smallest (inclusion) cliques with this property – **minimal critical sets** (MCS)

How to find all minimal critical sets?

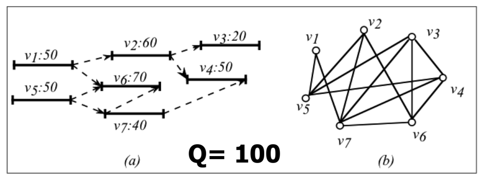
- index all nodes (in any order)
- for each node, explore in the DFS style all cliques containing this node and the nodes with smaller indexes
- all cliques exceeding the resource capacity are remembered (and not further extended)

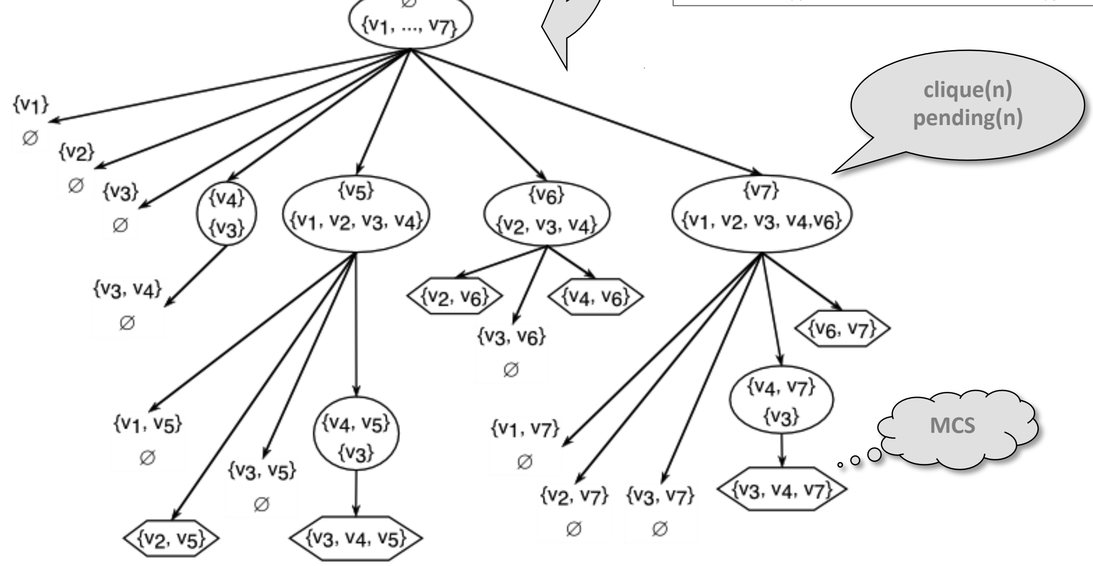
MCS-expand(p) — *so-far found part of clique*
  for each $v_i \in$ pending(p) do — *pending candidates to be included in the clique (they are connected with every node in p)*
    add a new node m_i successor of p
    pending(m_i) $\leftarrow \{v_j \in$ pending(p) $\mid j < i$ and $(v_i, v_j) \in E\}$ — *not finding identical cliques*
    clique(m_i) $\leftarrow$ clique(p) $\cup \{v_i\}$
    if clique(m_i) is over-consuming then MCS $\leftarrow$ MCS $\cup$ clique(m_i)
    else if pending(m_i) $\neq \emptyset$ then MCS-expand(m_i)
end

- The algorithm starts with a clique found so far (at the beginning it is empty) and a set of pending candidates to be included in the clique (at the beginning it is empty).
- We look for possible extensions of the clique by a node v_i (and then nodes with index smaller than i).

Resource conflict detection: an example

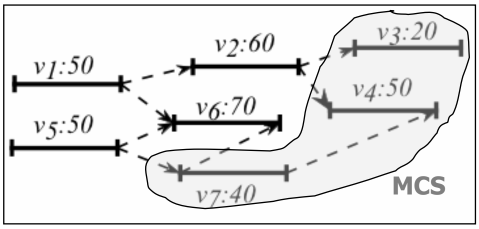
MCS-expand(p)
  for each $v_i \in$ pending(p) do
    add a new node m_i successor of p
    pending(m_i) $\leftarrow \{v_j \in$ pending(p) $\mid j < i$ and $(v_i, v_j) \in E\}$
    clique(m_i) $\leftarrow$ clique(p) $\cup \{v_i\}$
    if clique(m_i) is over-consuming than MCS $\leftarrow$ MCS $\cup$ clique(m_i)
    else if pending(m_i) $\neq \emptyset$ than MCS-expand(m_i)
end





How to remove a resource conflict?

- Let U= {z@[t_i,t_i'):q_i | i∈I} be a minimal critical set then any temporal constraint t_i'< t_j for i,j∈I removes the resource conflict.
  - this constraint removes edge (i,j) from the graph so U is no more a clique
  - any larger clique U': U⊆U' is no more a clique
  - no smaller clique U': U'⊆U was conflicting
- Some of suggested temporal **constraints** can be **in temporal conflict** with other constraints.
  - Example: t_4'<t_7 is in conflict with t_7'<t_4' and t_7<t_7'
  - Such resolvers are not used!
- Some suggested constraints are **too strong** (force removal of other edges from the graph).
  - Example: t_4'<t_3 is too strong as it forces t_7'<t_3 (via t_7'<t_4')
  - The planning algorithm will select one resolver to repair MCS so it is better to use only the necessary resolvers so they do not force other resolvers.




```
CPR(Φ, G, K, M, π)
   if G = K = M = ∅ then return(π)
   perform the three following steps in any order
      if G ≠ ∅ then do
         select any α ∈ G
         if θ(α/Φ) ≠ ∅ then return(CPR(Φ, G − {α}, K ∪ θ(α/Φ), M, π))
         else do
            relevant ← {a | a applicable to Φ and has a provider for α}
            if relevant = ∅ then return(failure)
            nondeterministically choose a ∈ relevant
            M′ ← the update of M with respect to Φ ∪ (F(a), C(a))
            return(CPR(Φ∪(F(a),C(a)), G∪F(a), K∪{θ(a/Φ)}, M′, π∪{a}))
      if K ≠ ∅ then do
         select any C ∈ K
         threat-resolvers ← {φ ∈ C | φ consistent with Φ}
         if threat-resolvers = ∅ then return(failure)
         nondeterministically choose φ ∈ threat-resolvers
         return(CPR(Φ ∪ φ, G, K − C, M, π))
      if M ≠ ∅ then do
         select U ∈ M
         resource-resolvers ← {φ resolver of U | φ is consistent with Φ}
         if resource-resolvers = ∅ then return(failure)
         nondeterministically choose φ ∈ resource-resolvers
         M′ ← the update of M with respect to Φ ∪ φ
         return(CPR(Φ ∪ φ, G, K, M′, π))
end
```

We just extent the algorithm for **planning with chronicles** to work with minimal conflict sets (in M) to resolve resource conflicts

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