Planning & Scheduling

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Course questions

What is the content?

- planning and scheduling
- but what is planning and scheduling?

Why could it be interesting to me?

- is it used somewhere?
- any applications?

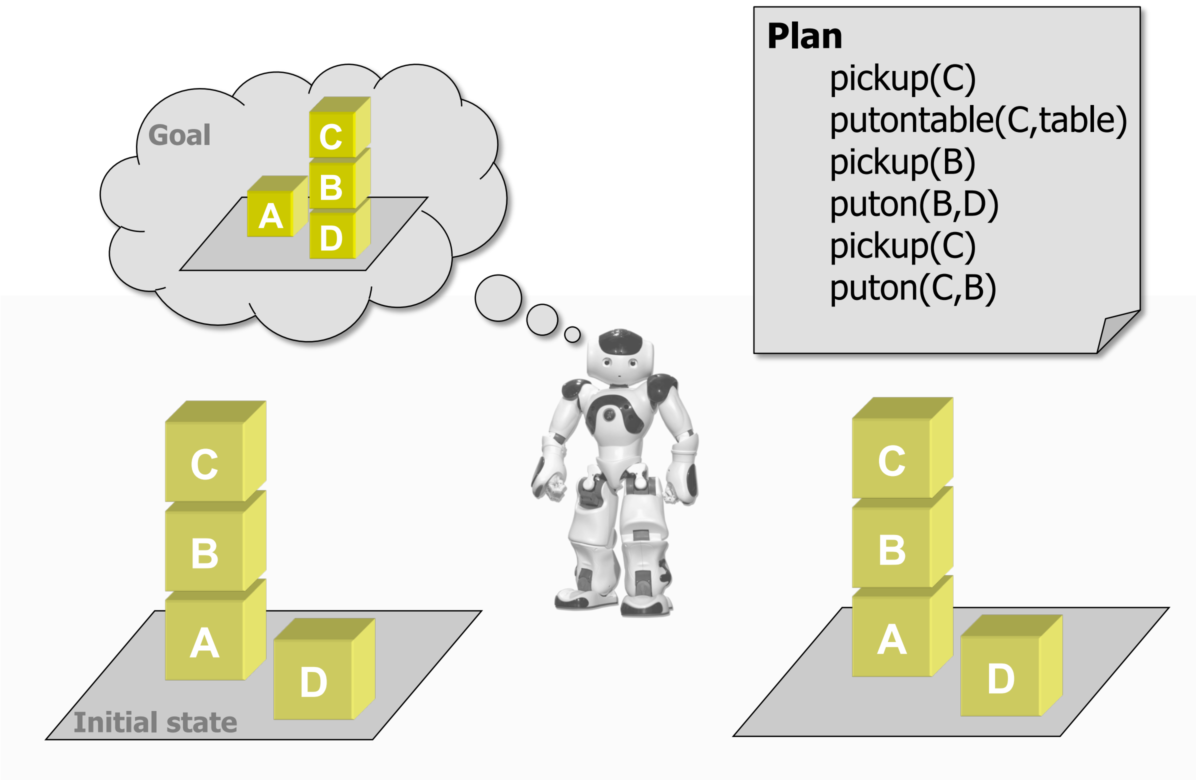
What is the course about?

- problem formalisation and modelling
- solving approaches

What?

What is planning and scheduling?
What is a difference between them?

What is planning?



Input:

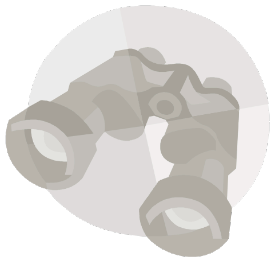
- – initial (current) state of the world
- – description of actions that can change the world
- – desired state of the world

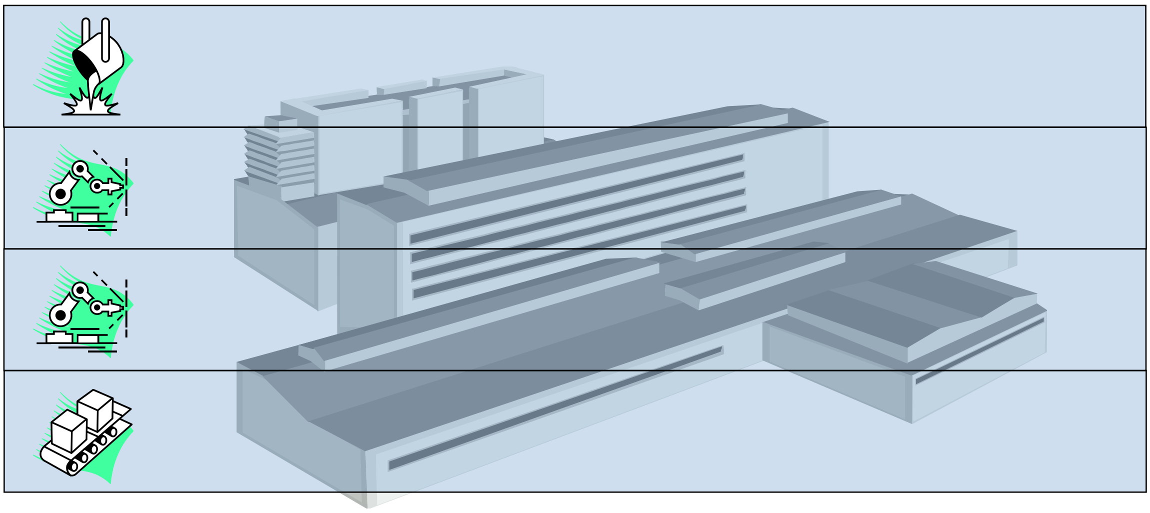
Output:

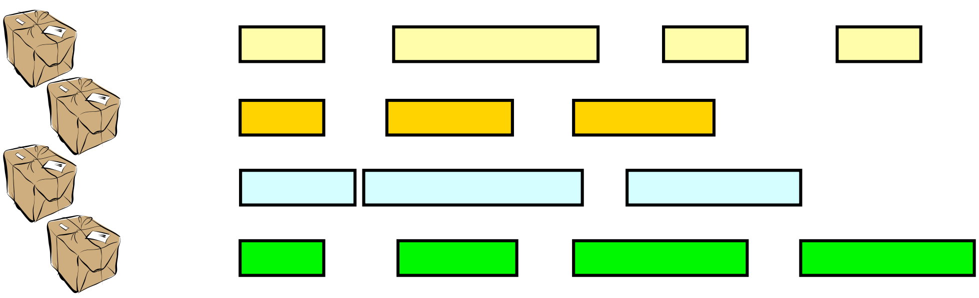
- – a sequence of actions (a plan)

Properties:

- – actions in the plan are unknown
- – time and resources are not assumed







Scheduling task at glance

Input:

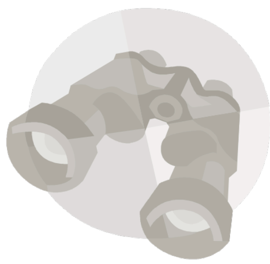
– a set of partially ordered activities

– available resources (machines, people, …)

Output:

– allocation of activities to time and resources (schedule)

- **Properties:**
  – activities are known in advance
  – limited time and resources



Planning and scheduling

Planning

– deciding which actions are necessary to achieve the goals

– topic of artificial intelligence

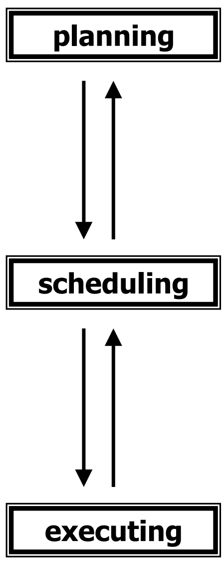
– complexity is usually worse than NP-c (in general, undecidable)

Scheduling

– deciding how to process the actions using given restricted resources and time

– topic of operations research

– complexity is typically NP-c



Why?

Is this technology practically useful?
Any applications?

Aircraft assembly

570 tasks, 17 resources
A traditional approach:
- ARTEMIS
- 20 hours to produce a schedule

Intelligent Planning and Scheduling:
- ARTEMIS substituted by a CSP
- 30 minutes to generate an optimal schedule
- 10 - 15% shorter makespan

Savings:
- 4 to 6 days shorter scheduled
- **$200k – $1m per day**

Contribution of On Time Systems

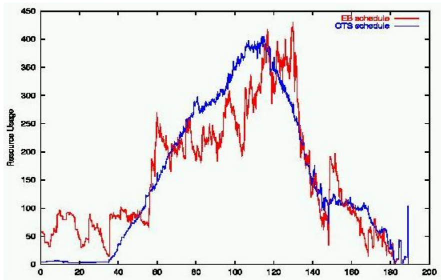
7000 tasks per boat and approx. 125 resource classes

A traditional approach:

- – ARTEMIS
- – 6 weeks to generate a schedule
- – very non-uniform resource profile

Intelligent Planning and Scheduling:

- – ARTEMIS substituted by a CSP
- – 2 days per schedule
- – uniform resource profile



Savings:

- **– 30% less overtime and sub-contracts**

Gulf war 1991:

A traditional approach:

- hundreds of human planners
- months to generate a plan

Intelligent Planning and Scheduling:

- System O-PLAN2

Savings:

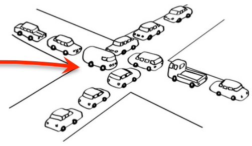
- faster background creation
- less flight missions
- Financial backflow >> **all research AI supported by US government**:
  - – since 1956
  - – not only IP&S, but **but all AI research!**

Launch: October 24, 1998

Target: Comet Borrelly

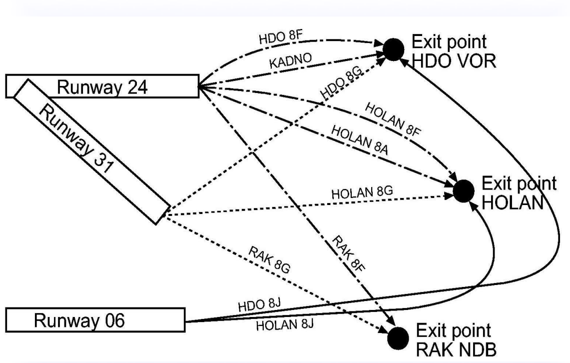
testing a payload of 12 advanced, high risk technologies

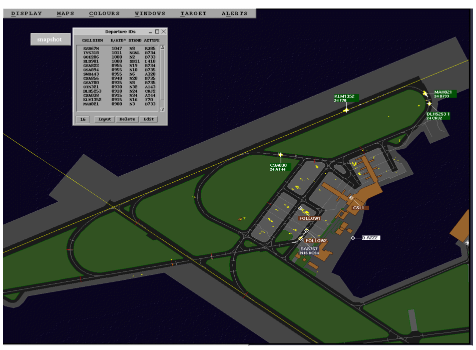
- **autonomous remote agent**
  - planning, execution, and monitoring spacecraft activities based on general commands from operators
  - three testing scenarios
    - 12 hours of low autonomy (execution and monitoring)
    - 6 days of high autonomy (operating camera, simulation of faults)
    - 2 days of high autonomy (keep direction)
      - » **beware of backtracking!**
      - » **beware of deadlock in plans!**



Departure management

- pre-flight control
  - exit assignment and clearance
  - coordinates with Brussels
- ground control
  - taxiing
- control tower
  - runway assignment
  - separation





MANTEA

(**MAN**agement of surface **T**raffic in **E**uropean **A**irports)

- implemented in **ILOG Scheduler**
- tested in **Prague** (27.5. – 7.6. 2002)

Contribution of NLR

About what?

What does this course bring?

Which topics are covered?

Course outline

Preliminaries

– search algorithms, constraint satisfaction and SAT

Planning

– classical planning (STRIPS)

– neo-classical planning (Graphplan)

– hierarchical planning

– planning with time and resources

Scheduling

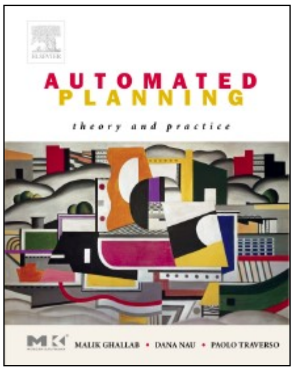
– classical scheduling

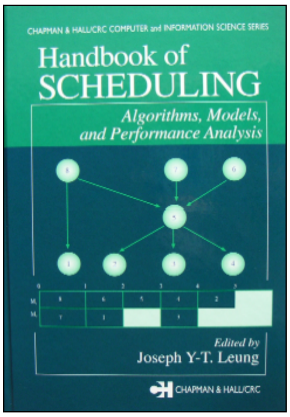
– constraint-based scheduling

Applications

Automated Planning: Theory and Practice

- M. Ghallab, D. Nau, P. Traverso
- http://www.laas.fr/planning/
- Morgan Kaufmann



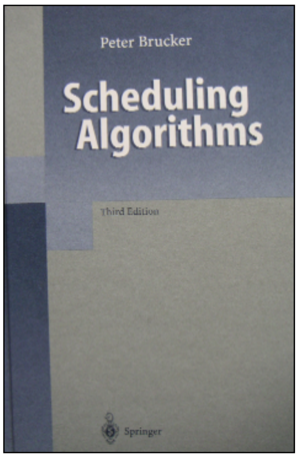


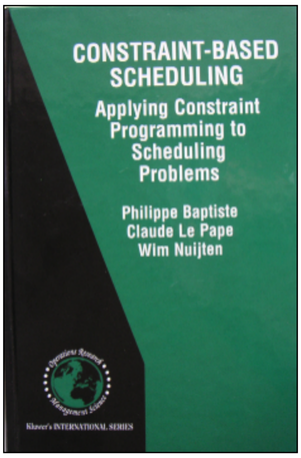
Handbook of Scheduling

- J. Leung
- Chapman&Hall/CRC

Scheduling Algorithms

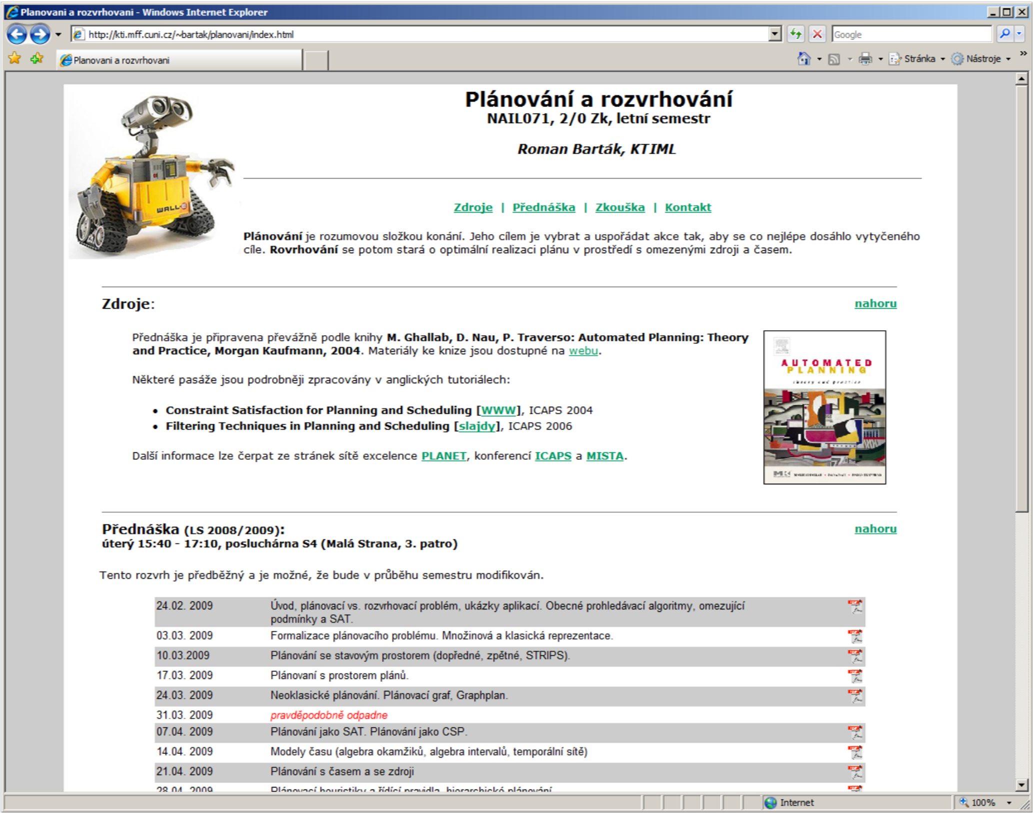
- P. Brucker
- Springer





Constraint-based Scheduling

- P. Baptiste, C. Le Pape, W. Nuijten
- Kluwer



Preliminaries

What am I supposed to know?

- search techniques
- basics of constraint satisfaction
- logic and SAT

Search techniques are the core solving approach used in AI (and beyond AI).

Classes of search techniques:

- **State-space search**
  - find a state (path to a state) with some properties
- **Problem-reduction search**
  - find a reduction of task to primitive tasks

soundness

- The output of the algorithm is a problem solution.

completeness

- If there is any solution then the algorithm finds it.

admissibility

- The algorithm guarantees finding an optimal solution.
- There must be some measure of optimality!
- It also means soundness and completeness.

State space S is a set of nodes (states) and the task is to find a state satisfying some goal condition g.

Formally, the **problem specification** is a triple (s_0,g,O):

- s_0 is the **initial state**
- g is a **goal condition** (the goal state satisfies g(s))
- O is a set of **operators** defining the next state
  - State space is defined recursively as:
    - $s_0 \in S$; if $s \in S$, $o \in O$ and o(s) is defined then $o(s) \in S$
  - o(s) is a child of node s

Breadth-First Search

explores tree levels

- q is a **queue**

```
bfs(s0,g,O)
    q ← {s0}
    while non-empty(q) do
        s ← first(q)
        if g(s) then return s
        q ← delete_first(q)
        q ← q + {s' | ∃o∈O, s'=o(s)}
    end while
    return failure
```

- **sound and complete**
- **Complexity** to find a goal node at depth d with the branching factor b:
  - time complexity $O(b^d)$
  - space complexity $O(b^d)$

Depth-first search

Depth-First Search

(backtracking)

go in one direction

backtrack upon failure

– q is a **stack**

```
dfs(s0,g,O)
    q ← {s0}
    while non-empty(q) do
        s ← first(q)
        if g(s) then return s
        q ← delete_first(q)
        q ← {s' | ∃o∈O, s'=o(s)} + q
    end while
    return failure
```

- **Sound and complete**, if there are no infinite branches or can be detected.
- **Complexity** to find a goal node at depth d:
  - Time complexity depends on teh selected direction (can explore a complete search space but can also go directly to the goal)
  - space complexity O(d)

Best-first search

Sometimes we are looking for a goal state while minimizing an objective function f(s).

Best-First Search

Go to the best next state

– q is a **priority queue**

```
bestfs(s0,g,O,f)
    q ← {s0}
    while non-empty(q) do
        s ← best(q,f)
        if g(s) then return s
        q ← delete_best(q,f)
        q ← q ⊕ {s' | ∃o∈O, s'=o(s)}
    end while
    return failure
```

- If f is not decreasing ($s'=o(s) \Rightarrow f(s) \leq f(s')$), then the found solution is optimal. If the search space is finite then the algorithm is admissible.
- If there is some $\delta>0$ s.t. $s'=o(s) \Rightarrow f(s)+\delta \leq f(s')$, then the algorithm is admissible even for infinite search space.

Branch and bound

Another algorithm optimizing objective f.

Depth-First Branch-and-Bound Search

Explore “all” branches

and remember the best

– q is a **stack**

```
dfbbs(s0,g,O,f)
    s* ← dummy % f(dummy)=∞
    q ← {s0}
    while non-empty(q) do
        s ← first(q)
        q ← delete_first(q)
        if g(s) & f(s)<f(s*) then
            s* ← s
        else
            q ← {s‘ | ∃o∈O, s‘=o(s)}+q
        end if
    end while
    return s*
```

- If f is not decreasing and a state space is finite and with no loops, then the algorithm is.

Greedy search

Greedy Search

Like DFS

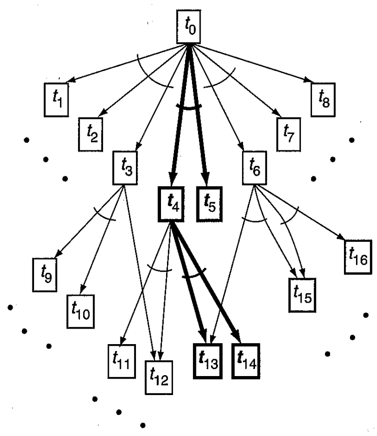
but no backtracks

```
gs(s0,g,O,f)
    s ← s0
    while not g(s) do
        s ← best({s‘ | ∃o∈O, s‘=o(s)},f)
    end while
    return s
```

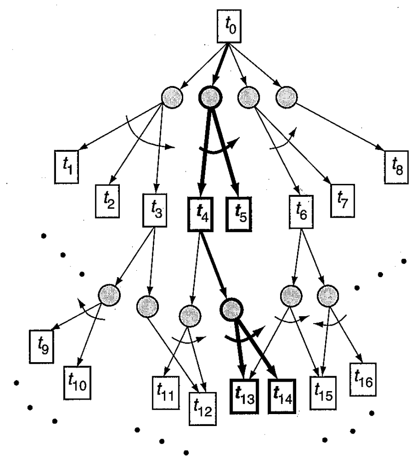
- No guarantee of optimality
- Sometimes saves a lot of time necessary to prove optimum.
- Frequently used to find the first solution.

Sometimes the operator o gives a set of children, **sub-problems**, and solution of them represents a portion of the solution of the parent.

This gives an **AND-OR graph**.



↳ **Problem-reduction search**



Problem Reduction Search

Decompose the problem

and find solutions of sub-problems

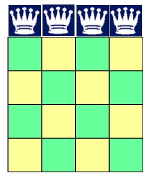
- non-deterministic
- naive
  - Repeatedly solves common sub-problems

```
preds(s,g,O)
    if g(s) then return s
    applicable ← {o∈O | o(s)↓}
    if applicable=∅ then return failure
    o ← choose_nondet(applicable)
    {s1,…,sn} ← o(s)
    for every si∈{s1,…,sn} do
        vi ← preds(si,g,O)
        if vi=failure then return failure
    end for
    return {v1,…,vn}
```

Constraint satisfaction

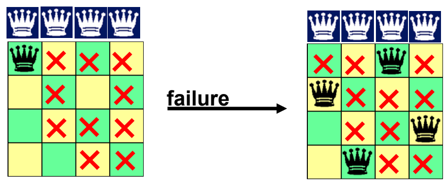
Modeling (problem formulation)

- N queens problem
- **decision variables** for positions of queens in rows
  r(i) in {1,…,N}
- **constraints** describing (non-)conflicts
  $\forall i \neq j \quad r(i) \neq r(j) \;\&\; |i-j| \neq |r(i)-r(j)|$



Search and inference (propagation)

- **backtracking** (assign values and return upon failure)
- infer consequences of decisions
  via maintaining **consistency**
  of constraints



Constraint satisfaction problem

based on **declarative problem description** via:

- **variables with domains** (sets of possible values)
  describe **decision points** of the problem with possible
  options for the decisions
  e.g. the start time of activity with time windows
- **constraints** restricting combinations of values,
  describe arbitrary **relations** over the set of variables
  e.g. end(A) < start(B)

A **feasible solution** to a constraint satisfaction problem is a complete assignment of variables satisfying all the constraints.

An **optimal solution** to a CSP is a feasible solution minimizing/maximizing a given objective function.

Search is combined with filtering techniques that prune the search space.

Maintaning Arc Consistency During Search

```
procedure labeling(V,D,C)
        if all variables from V are assigned then return V
        select not-yet assigned variable x from V
        for each value v from D_x do
                (TestOK,D') ← consistent(V,D,C∪{x=v})
                if TestOK=true then R ← labeling(V,D',C)
                        if R ≠ fail then return R
        end for
        return fail
end labeling
```


A formal system consisting of three constituents:

- **language**
  (a set of possible statements called formulas)
  e.g. $p \rightarrow q$
- **semantics**
  (assigns a meaning to each statement)
  e.g. if both p and q are true then $p \rightarrow q$ *is true*
- **proof theory**
  (rules to transform statements and derive new statements)
  e.g. the modus ponens rule ($p, p \rightarrow q \vdash q$)

The language is a **set** P **of propositions** – defined inductively starting from an enumerable set of atomic propositions (propositional variables) P_0:

- if $p \in P_0$ then $p \in P$,
- if $p \in P$ then $\neg p \in P$,
- If $p \in P$ and $q \in P$ then $p \wedge q \in P$,
- Nothing else is a propositional formula.

- We can also define
  - $p \vee q$ as abbreviation for $\neg(\neg p \wedge \neg q)$
  - $p \rightarrow q$ as abbreviation for $\neg p \vee q$

- **Conjunctive Normal Form** (CNF):
  - **formula** is a conjunction of clauses
  - **clause** is a disjunction of literals (clause with a single literal is call a **unit clause**)
  - **literal** is a propositional variable (positive literal) or its negation (negative literal)

A model of propositional formula is an assignment of truth values to the propositional variables (interpretation) for which the formula evaluates to true:

- $\neg p$ is true if and only if p is not true
- $p \wedge q$ is true if and only if both p and q are true

A satisfiability problem (SAT) is the problem of determining whether a formula has a model.

- The SAT problem (given as a CNF) can be solved using **depth-first search** with **unit propagation**.
- **Unit propagation** determines the truth values of literals in unit clauses as follows:
  - the variable in a positive literal is assigned to true,
  - the variable in a negative literal is assigned to false

  The assigned value is propagated to other clauses as follows.

  If D is assigned to true then:

  - the clause containing D (e.g. $A \vee \neg B \vee D$) can be discarded
  - the clauses containing $\neg D$ (e.g. $C \vee \neg D \vee E$) can be simplified by removing $\neg D$ ($C \vee E$)

  Symmetrically for the case when D is assigned to false.

```
procedure DP(A, Assignment)
    A: is a CNF formula (represented as a set of clauses)
    A and Assignment are local within DP
    if ∅∈A then return
    if A=∅ then exit with Assignment
    Unit-Propagate(A, Assignment)
    select a variable P such that P or ¬P occurs in A
    DP(A∪{P},Assignment)
    DP(A∪{¬P},Assignment)
end DP
```

```
procedure Unit-Propagate(A, Assignment)
    A and Assignment are global within Unit-Propagate
    while there is a unit clause {l} in A do
        Assignment ← Assignment ∪ {l}
        for every clause C∈A do
            if l∈C then A ← A - {C}
            else if ¬l∈C then A ← A - {C} ∪ (C-{¬ l})
end Unit-Propagate
```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